Logarithmic Functions

Warm Up

Lesson Presentation

Lesson Quiz

Logarithmic Functions

Warm Up

Use mental math to evaluate.

1. 4^{-3} $\frac{1}{64}$

2. $16^{\frac{1}{4}}$ 2

3. 10^{-5} 0.00001

4. $\left(\frac{2}{3}\right)^{-3}$ $\frac{27}{8}$

5. A power has a base of –2 and exponent of 4. Write and evaluate the power.

$(-2)^4 = 16$

Objectives

Write equivalent forms for exponential and logarithmic functions.

Write, evaluate, and graph logarithmic functions.

Vocabulary

logarithm

common logarithm

logarithmic function

Logarithmic Functions

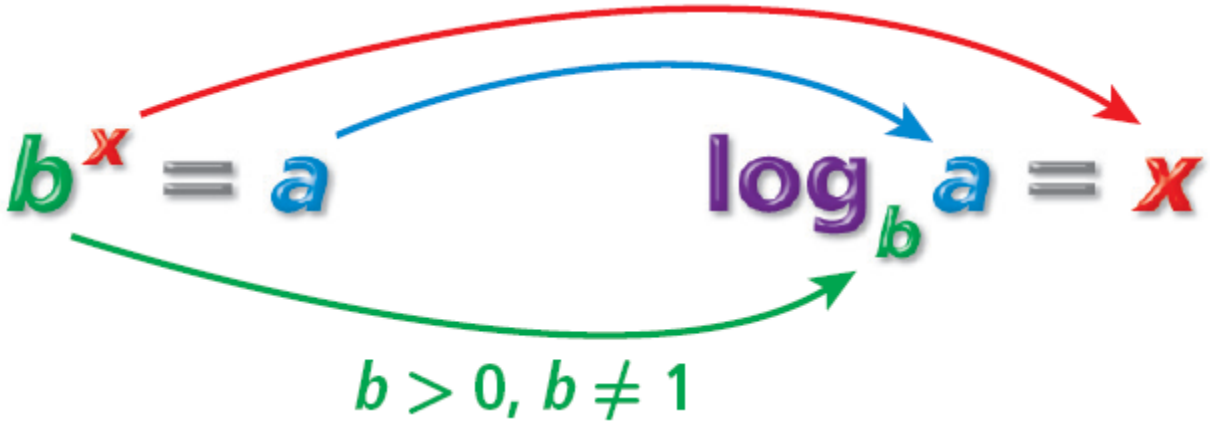
How many times would you have to double $1 before you had $8? You could use an exponential equation to model this situation. $1(2^x) = 8$. You may be able to solve this equation by using mental math if you know $2^3 = 8$. So you would have to double the dollar 3 times to have $8.

Logarithmic Functions

How many times would you have to double $1 before you had $512? You could solve this problem if you could solve $2^x = 8$ by using an inverse operation that undoes raising a base to an exponent equation to model this situation. This operation is called finding the logarithm. A **logarithm** is the exponent to which a specified base is raised to obtain a given value.

Logarithmic Functions

You can write an exponential equation as a logarithmic equation and vice versa.



Reading Math

Read $\log_b a = x$, as "the log base b of a is x."
Notice that the log is the exponent.

Logarithmic Functions

Example 1: Converting from Exponential to Logarithmic Form

Write each exponential equation in logarithmic form.

Exponential Equation	Logarithmic Form
$3^5 = 243$	$\log_3 243 = 5$
$25^{\frac{1}{2}} = 5$	$\log_{25} 5 = \frac{1}{2}$
$10^4 = 10{,}000$	$\log_{10} 10{,}000 = 4$
$6^{-1} = \frac{1}{6}$	$\log_6 \frac{1}{6} = -1$
$a^b = c$	$\log_a c = b$

The base of the exponent becomes the base of the logarithm.

The exponent is the logarithm.

An exponent (or log) can be negative.

The log (and the exponent) can be a variable.

Logarithmic Functions

Check It Out! Example 1

Write each exponential equation in logarithmic form.

	Exponential Equation	Logarithmic Form
a.	$9^2 = 81$	$\log_9 81 = 2$
b.	$3^3 = 27$	$\log_3 27 = 3$
c.	$x^0 = 1 (x \neq 0)$	$\log_x 1 = 0$

The base of the exponent becomes the base of the logarithm.

The exponent of the logarithm.

The log (and the exponent) can be a variable.

Example 2: Converting from Logarithmic to Exponential Form

Write each logarithmic form in exponential equation.

Logarithmic Form	Exponential Equation
$\log_9 9 = 1$	$9^1 = 9$
$\log_2 512 = 9$	$2^9 = 512$
$\log_8 2 = \frac{1}{3}$	$8^{\frac{1}{3}} = 2$
$\log_4 \frac{1}{16} = -2$	$4^{-2} = \frac{1}{16}$
$\log_b 1 = 0$	$b^0 = 1$

The base of the logarithm becomes the base of the power.

The logarithm is the exponent.

A logarithm can be a negative number.

Any nonzero base to the zero power is 1.

Check It Out! Example 2

Write each logarithmic form in exponential equation.

Logarithmic Form	Exponential Equation
$\log_{10} 10 = 1$	$10^1 = 10$
$\log_{12} 144 = 2$	$12^2 = 144$
$\log_{\frac{1}{2}} 8 = -3$	$\left(\frac{1}{2}\right)^{-3} = 8$

The base of the logarithm becomes the base of the power.

The logarithm is the exponent.

An logarithm can be negative.

Logarithmic Functions

A logarithm is an exponent, so the rules for exponents also apply to logarithms. You may have noticed the following properties in the last example.

Special Properties of Logarithms

For any base b such that $b > 0$ and $b \neq 1$,

LOGARITHMIC FORM	EXPONENTIAL FORM	EXAMPLE
Logarithm of Base b		
$\log_b b = 1$	$b^1 = b$	$\log_{10} 10 = 1$ $10^1 = 10$
Logarithm of 1		
$\log_b 1 = 0$	$b^0 = 1$	$\log_{10} 1 = 0$ $10^0 = 1$

Logarithmic Functions

A logarithm with base 10 is called a **common logarithm**. If no base is written for a logarithm, the base is assumed to be 10. For example, $\log 5 = \log_{10} 5$.

You can use mental math to evaluate some logarithms.

Example 3A: Evaluating Logarithms by Using Mental Math

Evaluate by using mental math.

log 0.01

$10^{?} = 0.01$ *The log is the exponent.*

$10^{-2} = 0.01$ *Think: What power of 10 is 0.01?*

$\log 0.01 = -2$

Example 3B: Evaluating Logarithms by Using Mental Math

Evaluate by using mental math.

$\log_5 125$

$5^{?} = 125$ *The log is the exponent.*

$5^{3} = 125$ *Think: What power of 5 is 125?*

$\log_5 125 = 3$

Example 3C: Evaluating Logarithms by Using Mental Math

Evaluate by using mental math.

$\log_5 \frac{1}{5}$

$5^{?} = \frac{1}{5}$ *The log is the exponent.*

$5^{-1} = \frac{1}{5}$ *Think: What power of 5 is $\frac{1}{5}$?*

$\log_5 \frac{1}{5} = -1$

Check It Out! Example 3a

Evaluate by using mental math.

log 0.00001

$10^{?} = 0.00001$ *The log is the exponent.*

$10^{-5} = 0.01$ *Think: What power of 10 is 0.01?*

$\log 0.00001 = -5$

Check It Out! Example 3b

Evaluate by using mental math.

$\log_{25} 0.04$

$25^{?} = 0.04$ *The log is the exponent.*

$25^{-1} = 0.04$ *Think: What power of 25 is 0.04?*

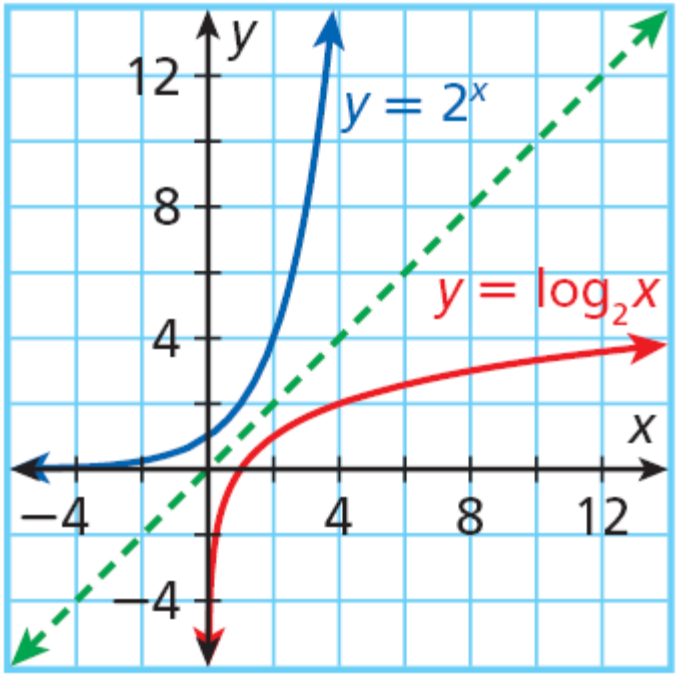
$\log_{25} 0.04 = -1$

Logarithmic Functions

Because logarithms are the inverses of exponents, the inverse of an exponential function, such as $y = 2^x$, is a **logarithmic function**, such as $y = \log_2 x$.

You may notice that the domain and range of each function are switched.

The domain of $y = 2^x$ is all real numbers (R), and the range is $\{y|y > 0\}$. The domain of $y = \log_2 x$ is $\{x|x > 0\}$, and the range is all real numbers (R).

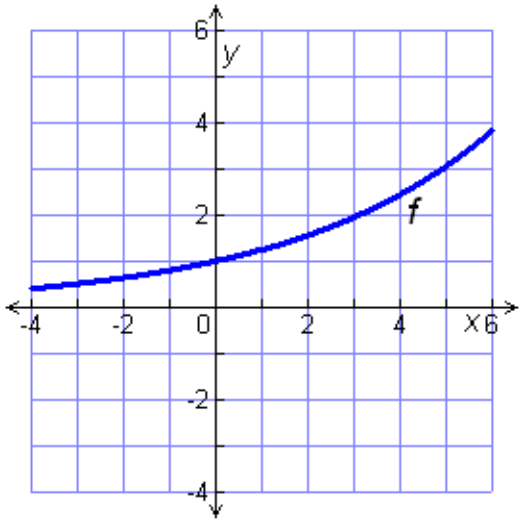


Example 4A: Graphing Logarithmic Functions

Use the *x*-values {−2, −1, 0, 1, 2}. Graph the function and its inverse. Describe the domain and range of the inverse function.

$f(x) = 1.25^x$

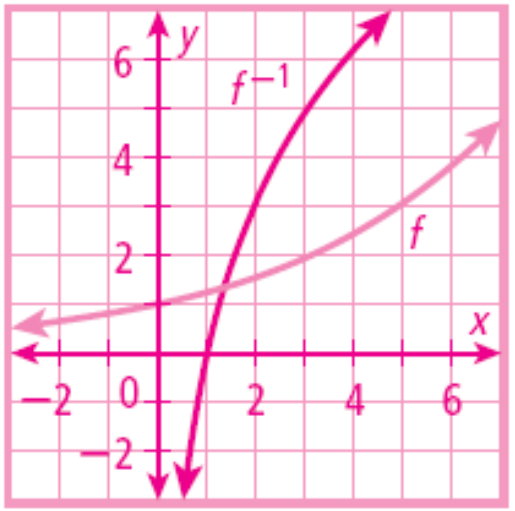
Graph $f(x) = 1.25^x$ by using a table of values.



x	−2	−1	0	1	2
$f(x) = 1.25^x$	0.64	0.8	1	1.25	1.5625

Example 4A Continued

To graph the inverse, $f^{-1}(x) = \log_{1.25} x$, by using a table of values.



x	0.64	0.8	1	1.25	1.5625
$f^{-1}(x) = \log_{1.25} x$	−2	−1	0	1	2

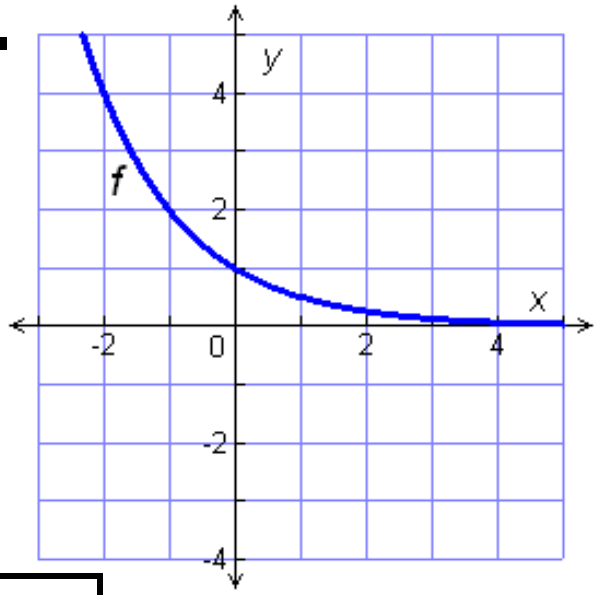
The domain of $f^{-1}(x)$ is $\{x | x > 0\}$, and the range is R.

Example 4B: Graphing Logarithmic Functions

Use the *x*-values {−2, −1, 0, 1, 2}. Graph the function and its inverse. Describe the domain and range of the inverse function.

$$f(x) = \left[\frac{1}{2}\right]^x$$

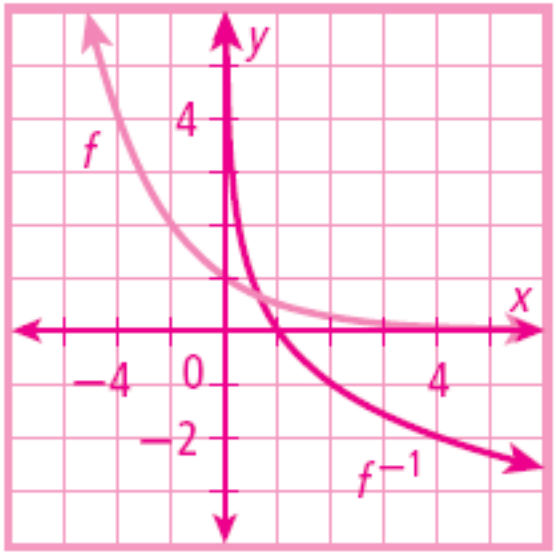
Graph $f(x) = \frac{1}{2}^x$ by using a table of values.



x	−2	−1	0	1	2
$f(x) = (\frac{1}{2})^x$	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$

Example 4B Continued

To graph the inverse, $f^{-1}(x) = \log_{\frac{1}{2}} x$, by using a table of values.



x	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$
$f^{-1}(x) = \log x$	−2	−1	0	1	2

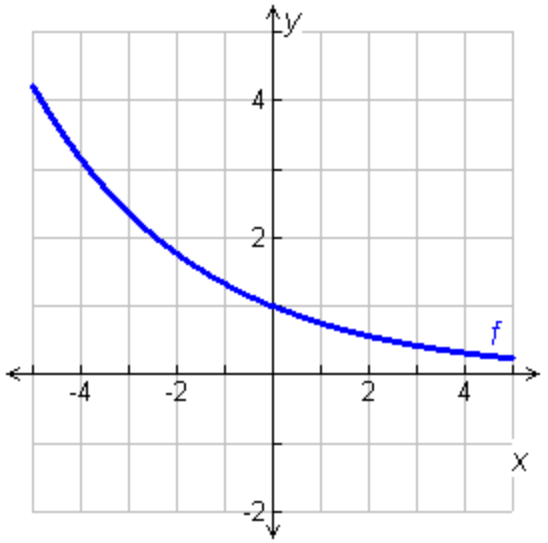
The domain of $f^{-1}(x)$ is $\{x|x > 0\}$, and the range is R.

Logarithmic Functions

Check It Out! Example 4

Use $x = -2, -1, 1, 2,$ and 3 to graph $f(x) = \left(\frac{3}{4}\right)^x$. Then graph its inverse. Describe the domain and range of the inverse function.

Graph $f(x) = \left(\frac{3}{4}\right)^x$ by using a table of values.

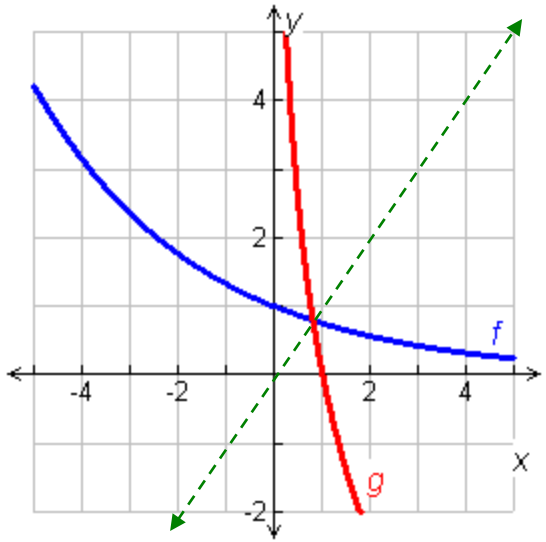


x	–2	–1	1	2	3
$f(x) = \frac{3}{4}^x$	$\frac{16}{9}$	$\frac{4}{3}$	$\frac{3}{4}$	$\frac{9}{16}$	$\frac{27}{64}$

Logarithmic Functions

Check It Out! Example 4

To graph the inverse, $f^{-1}(x) = \log_{\frac{3}{4}} x$, by using a table of values.



x	$\frac{16}{9}$	$\frac{4}{3}$	$\frac{3}{4}$	$\frac{9}{16}$	$\frac{27}{64}$
$f^{-1}(x) = \log_{\frac{3}{4}} x$	–2	–1	1	2	3

The domain of $f^{-1}(x)$ is $\{x|x > 0\}$, and the range is R.

Logarithmic Functions

Helpful Hint

The LOG key is used to evaluate logarithms in base 10. 2nd 10^x LOG is used to find 10^x, the inverse of log.

Example 5: Food Application

The table lists the hydrogen ion concentrations for a number of food items. Find the pH of each.

Substance	H^+ conc. (mol/L)
Milk	0.00000025
Tomatoes	0.0000316
Lemon juice	0.0063

Example 5 Continued

Milk

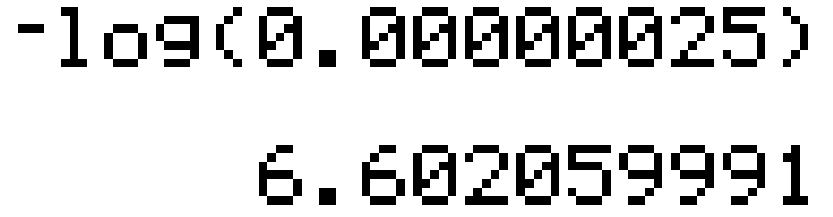
The hydrogen ion concentration is 0.00000025 moles per liter.

$pH = -\log[H^+]$

$pH = -\log(0.00000025)$

Substitute the known values in the function.

Use a calculator to find the value of the logarithm in base 10. Press the LOG key.



Milk has the pH of about 6.6.

Example 5 Continued

Tomatoes

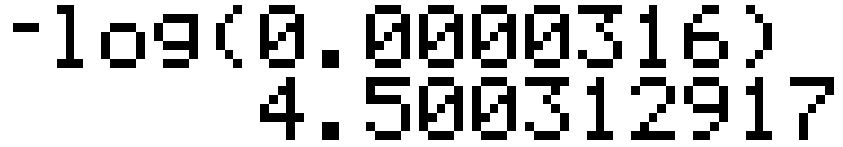
The hydrogen ion concentration is 0.0000316 moles per liter.

$pH = -\log[H^+]$

$pH = -\log(0.0000316)$

Substitute the known values in the function.

Use a calculator to find the value of the logarithm in base 10. Press the LOG key.



Tomatoes have the pH of about 4.5.

Example 5 Continued

Lemon juice

The hydrogen ion concentration is 0.0063 moles per liter.

$pH = -\log[H^+]$

$pH = -\log(0.0063)$

Substitute the known values in the function.

Use a calculator to find the value of the logarithm in base 10. Press the LOG key.

Lemon juice has the pH of about 2.2.

Check It Out! Example 5

What is the pH of iced tea with a hydrogen ion concentration of 0.000158 moles per liter?

The hydrogen ion concentration is 0.000158 moles per liter.

$pH = -\log[H^+]$

$pH = -\log(0.000158)$

Substitute the known values in the function.

Use a calculator to find the value of the logarithm in base 10. Press the LOG key.

Iced tea has the pH of about 3.8.

Logarithmic Functions

Lesson Quiz: Part I

1. Change $6^4 = 1296$ to logarithmic form. $\log_6 1296 = 4$

2. Change $\log_{27} 9 = \frac{2}{3}$ to exponential form. $27^{\frac{2}{3}} = 9$

Calculate the following using mental math.

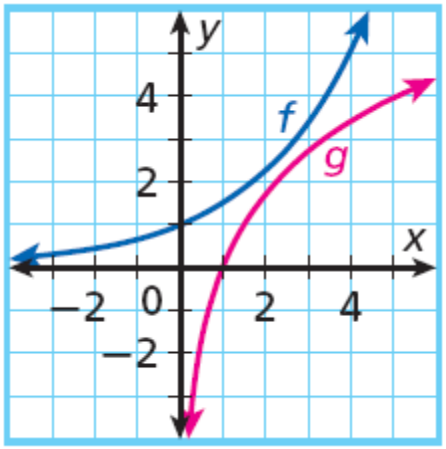
3. log 100,000 5

4. $\log_{64} 8$ 0.5

5. $\log_3 \frac{1}{27}$ −3

Lesson Quiz: Part II

6. Use the x-values $\{-2, -1, 0, 1, 2, 3\}$ to graph $f(x) = \left(\frac{5}{4}\right)^x$. Then graph its inverse. Describe the domain and range of the inverse function.



D: $\{x > 0\}$; R: all real numbers