Name ______________________________ Date ______________ Class______________

LESSON 13-1

Problem Solving

Complex Numbers and Roots

At a carnival, a new attraction allows contestants to jump off a springboard onto a platform to be launched vertically into the air. The object is to ring a bell located 20 feet overhead. The distance from the bell in feet is modeled by the function $dt = 16t^2 - bt + 20$, where t is the time in seconds after leaving the platform, and b is the takeoff velocity from the platform.

1. Kate watches some of the contestants. She theorizes that if the platform launches a contestant with a takeoff velocity of at least 32 feet per second, the contestant can ring the bell.
   a. Find the zeros for the function using 32 feet per second as the takeoff velocity. ______________________________
   b. Is Kate's theory valid? Explain.

   ____

2. Mirko suggests they vary the value of *b* and determine for which values of *b* the roots are real.
   a. Complete the table to show the roots for different values of *b.*
   b. For which values of *b* in the table are the roots real?

   ____
   c. What difference does it make if the roots are real?

b	Function	Roots
24	$d(t) = 16t^2 - 24t + 20$	
32	$d(t) = 16t^2 -$ __$t + 20$	
40	$d(t) = 16t^2 -$ __$t + 20$	
48	$d(t) = 16t^2 -$ __$t + 20$	

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3. Using the results from the table, and the function, estimate the minimum takeoff velocity needed for a contestant to be able to ring the bell. ______________________________

Choose the letter for the best answer.

4. Mirko suggests using four bells at heights of 15, 20, 25, and 30 feet from the platform. How many of the bells can a contestant reach if the takeoff velocity is 32 feet per second?
   A 3 C 1
   B 2 D 0

5. At what height must a bell be placed for a contestant to reach it with a takeoff velocity of 48 feet per second?
   A 20 feet or less
   B 25 feet or less
   C 30 feet or less
   D 36 feet or less

13. a. The beginning and end of the flight when the speed of the rocket is 0

b. $t = -3 \pm 5i$

c. No; possible answer: the zeros are imaginary because the graph never crosses the x-axis so the function never equals 0. The speed of the rocket must be 0 before takeoff and after landing.

Reteach

1. $6i\sqrt{2}$
2. $12i\sqrt{2}$
3. $10i$
4. $15i\sqrt{6}$
5. $16i$
6. $-7i\sqrt{2}$
7. $9i$
8. $1 - 4i$
9. $12 + i$
10. $x = \pm 3i\sqrt{2}$
11. $x = \pm\sqrt{-4}$
    $x = \pm 2i$
12. $x^2 = -49$
    $x = \pm\sqrt{-49}$
    $x = \pm 7i$
13. $x^2 = -100$
    $x = \pm\sqrt{-100}$
    $x = \pm 10i$
14. $x^2 = -36$
    $x = \pm\sqrt{-36}$
    $x = \pm 6i$
15. $x^2 = -12$
    $x = \pm\sqrt{(4)(3)(-1)}$
    $x = \pm 2i\sqrt{3}$

Challenge

1. $2i, -7i$
2. $-6i, -8i$
3. $9i, -12i$
4. $5i, 49i$
5. $-16i, -36i$
6. $-\frac{4}{3}i, -i$
7. $\frac{1}{5}i, 2i$
8. $-3i, -5i$
9. $-\frac{13}{4}i, 5i$
10. $\frac{5}{6}i, \frac{3}{2}i$

Problem Solving

1. a. $t = 1 \pm \frac{i}{2}$

b. No; possible answer: the roots are imaginary numbers.

2. a.

b	Function	Roots
24	$d(t) = 16t^2 - 24t + 20$	$\frac{1}{4}\left(3 \pm i\sqrt{11}\right)$
32	$d(t) = 16t^2 - \_\_t + 20$	$1 \pm \frac{i}{2}$
40	$d(t) = 16t^2 - \_\_t + 20$	$\frac{1}{4}\left(5 \pm \sqrt{5}\right)$
48	$d(t) = 16t^2 - \_\_t + 20$	$\frac{3}{2} \pm \sqrt{1}$

b. $b = 40$ and 48

c. Possible answer: Real roots mean that ringing the bell is possible.

3. About 36 feet per second
4. C
5. D

Reading Strategies

1. $i\sqrt{9}$; $3i$; $\sqrt{-8}$
2. $3i$ or $-3i$
3. a. $x^2 = -1$, so $x = \pm\sqrt{-1}$, $x = i$ and $-i$

   b. Because the square of a real number cannot be a negative number
4. $\left(\sqrt{5}i\right)^2 = \left(\sqrt{5}\right)^2 (i)^2 = 5(-1) = -5; \left(-\sqrt{5}i\right)^2$
   $= \left(-\sqrt{5}\right)^2 (i)^2 = 5(-1) = -5$
5. Real number; $(3i)(5i) = 15i^2 = 15(-1)$
   $= -15$