Name ______________________________ Date ______________ Class______________

LESSON 13-2

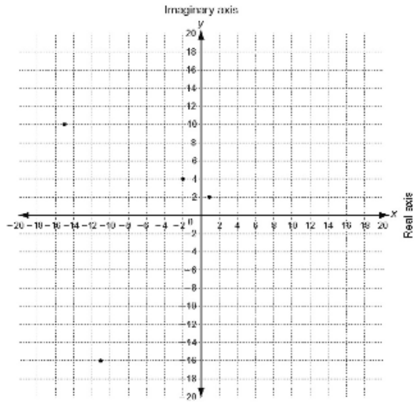
Problem Solving

Operations with Complex Numbers

Hannah and Aoki are designing fractals. Aoki recalls that many fractals are based on the Julia Set, whose formula is $Z_{n+1} = (Z_n)^2 + c$, where c is a constant. Hannah suggests they make their own fractal pattern using this formula, where $c = 1$ and $Z_1 = 1 + 2i$.

1. Complete the table to show values of n and Z_n.

n	$Z_{n+1} = (Z_n)^2 + c$	Z_n
1	$Z_1 = 1 + 2i$	$Z_1 = 1 + 2i$
2	$Z_2 = (1 + 2i)^2 + 1$	$Z_2 =$
3	$Z_3 =$ (__________)$^2 + 1$	$Z_3 =$
4	$Z_4 =$ (_____________)$^2 + 1$	$Z_4 =$



2. Four points are shown on the complex plane. Which point is not part of the fractal pattern they have created? Explain.

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Choose the letter for the best answer.

3. Aoki creates a second pattern by changing the value of c to 3. What happens to Z_n as n increases?
   - A The imaginary part is always twice the real part.
   - B The real and imaginary parts become equal.
   - C The real part becomes zero.
   - D The imaginary part becomes zero.

4. Hannah changes the formula to $Z_{n+1} = \dfrac{1}{(Z_n)^2} + c.$ Leaving $c = 1$ and $Z_1 = 1 + 2i$, what is the value of Z_2?
   - A $0.48 - 0.16i$
   - B $0.88 - 0.16i$
   - C $1.2 - 0.4i$
   - D $2.2 - 0.4i$

5. Aoki takes Hannah's new formula, leaves $c = 1$, and sets $Z_1 = \dfrac{1}{1+2i}$. What is the value of Z_3?
   - A $Z_3 = -11 - 16i$
   - B $Z_3 = 2 + 2i$
   - C $Z_3 = 0.48 - 0.16i$
   - D $Z_3 = 147.4 + i$

6. Hannah reverts to $Z_{n+1} = (Z_n)^2 + c$. She sets $Z_1 = i$ and $c = i$. Which statement is NOT true?
   - A Z_n flip-flops between $(-1 + i)$ and $(-i)$.
   - B The coefficient of i never reaches 2.
   - C The imaginary part becomes zero.
   - D On a graph $Z_1 - Z_3$ create a triangle.

3. The answers are the same. Yes; this will always be true in the system of real numbers. The order of operations can be reversed in this case; yes
4. $\sqrt{3}i \cdot \sqrt{12}i = \sqrt{3} \cdot \sqrt{12} \cdot i^2 = \sqrt{36}i^2$
$= 6 \cdot -1 = -6; \sqrt{36} = 6$
5. The answers are different. The order of operations cannot be changed in this case.
6. Possible answer: When multiplying radicals that have negative radicands, first simplify the radical using the imaginary number *i,* and then find the product.
7. –32
8. 12
9. –5
10. $-30i$
11. $6i$
12. $4i\sqrt{2}$

Problem Solving

1.

n	$Z_{n+1} = (Z_n)^2 + c$	Z_n
1	$Z_1 = 1 + 2i$	$Z_1 = 1 + 2i$
2	$Z_2 = (1 + 2i)^2 + 1$	$Z_2 = -2 + 4i$
3	$Z_3 = (-2 + 4i)^2 + 1$	$Z_3 = -11 - 16i$
4	$Z_4 = (-11 - 16i)^2 + 1$	$Z_4 = -134 + 352i$

2. $(-13, -35i)$; possible answer: this point cannot be generated using the given formula.
3. D
4. B
5. A
6. C

Reading Strategies

1. a. $-3i$
   b. $3 - 4i$
   c. $-1 - i$
   d. $-3 - 4i$
   e. $3i$
   f. $2 + i$
   g. –4
2. $5 + \sqrt{-4} = 5 + 2i = 5 + 2i$; located 5 units to the right and two units up
3. $\sqrt{2}$; the point $(-1 + i)$ is one vertex of a right triangle with vertices at the origin and $(-1 + 0i)$. Each leg of the triangle equals 1. Using the Pythagorean Theorem, $1^2 + 1^2 = c^2$, $c^2 = 2$, $c = \sqrt{2}$.
4. The real value is the same for both, and $3i$ and $-3i$ are the same distance from the real number axis. So the distances to the origin are corresponding sides on congruent triangles.

Answers for Unit 5

14-1 FACTORING $x^2 + bx + c$

Practice A

1. 3; 2
2. 4; 1
3. 5; 4
4. $(x + 7)(x + 3)$
5. $(x + 6)(x + 5)$
6. $(x + 8)(x + 2)$
7. 6; 2
8. 5; 3
9. 16; 1
10. $(x - 9)(x - 3)$
11. $(x - 4)(x - 11)$
12. $(x - 8)(x - 5)$
13. 10; 4
14. 3; 1
15. 8; 4
16. 12; 2
17. 14; 2
18. 5; 2
19. $(x + 3)(x - 5)$
20. $(x + 2)(x - 10)$
21. $(x + 6)(x - 8)$
22. $(x + 3)(x - 4)$
23. $(x + 1)(x - 3)$
24. $(x + 1)(x - 2)$
25. 1; 5

n	$n^2 + 6n + 5$
0	$0^2 + 6(0) + 5 = 5$
1	$1^2 + 6(1) + 5 = 12$
2	$2^2 + 6(2) + 5 = 21$
3	$3^2 + 6(3) + 5 = 32$
4	$4^2 + 6(4) + 5 = 45$