Name ______________________________________ Date _________________ Class_________________

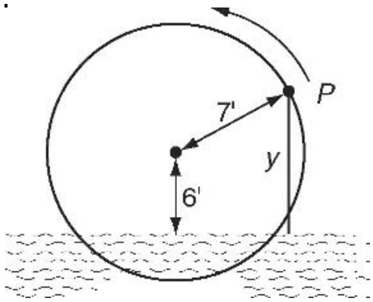
LESSON 11-1

Challenge

Sinusoidal Functions as Mathematical Models

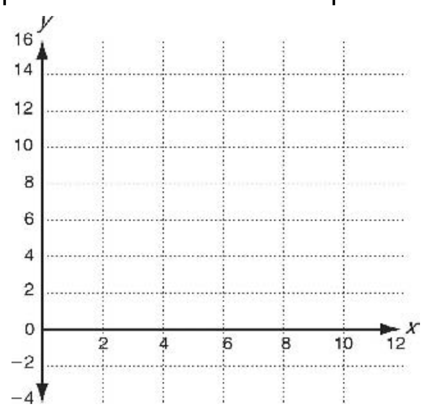
In many real-world situations, a dependent variable repeats its values at regular intervals as the independent variable changes. In such cases, a sine or cosine function would be a reasonable mathematical model.

As represented at right, a waterwheel with a radius of 7 feet rotates at 6 revolutions per minute. You start a stopwatch when the point *P* on the rim of the wheel is at its lowest possible position. After 5 seconds, point *P* is at its greatest height.



To write an equation that will model the distance, *y*, of point *P* from the surface of the water in terms of the number of seconds, *x*, on the stopwatch, assume that *y* varies sinusoidally with *x*, and begin with a sine equation.

1. In the general sine equation, $y = a \sin b(x - h) + k$, identify the constants that need to be determined. ________________________
2. Since the center is 6 feet above the water, the horizontal line of symmetry of the sine curve will be 6 units above the *x*-axis. This information gives you the value of one of the constants. Write the value of this constant. ________________________
3. Since the radius of the wheel is 7 feet, point *P* goes 7 feet above and below the center of the wheel. From this information, write the value of another constant. ________________________
4. The wheel makes 6 complete revolutions every 60 seconds. From this information, write the value of another constant. ________________________
5. *P* was at its maximum after 5 seconds. Use this information to write the value of the last constant. ________________________
6. Write the required equation to model the movement of the waterwheel. ____________________________________
7. Sketch the graph of your equation on the coordinate plane below.



8. Use your graph to predict the height of point *P* above the surface of the water when the stopwatch shows 7 seconds.

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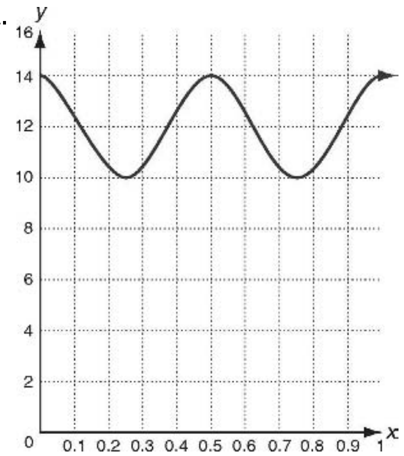
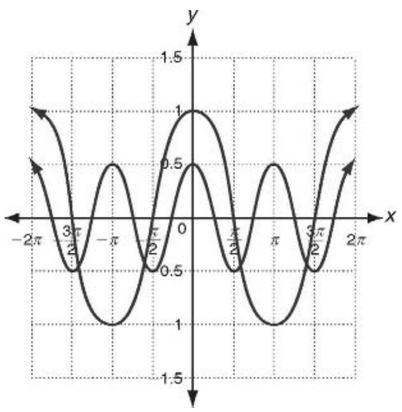
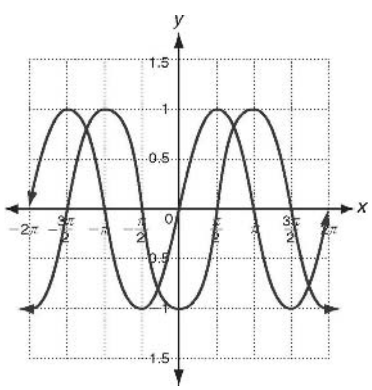
5. a. 

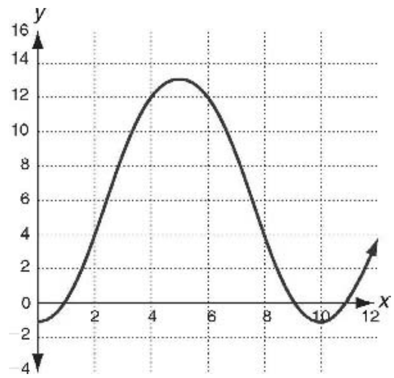
   b. 10 cm

Review for Mastery

1. 0.5
2. π
3. 0.5, –0.5
4. 1 cycle
5. 

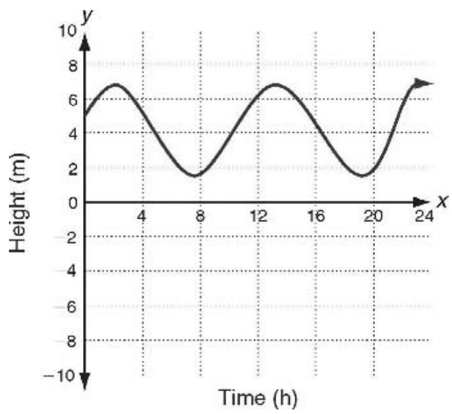
6. $h = \frac{\pi}{2}$; phase shift: $\frac{\pi}{2}$ radians to the right
7. $\frac{\pi}{2}, \frac{3\pi}{2}$
8. Maximum of 1 at π; minimum of –1 at 0 and 2π
9. 

Challenge

1. a, *b, h, k*
2. $k = 6$
3. $a = 7$
4. $b = \frac{\pi}{5}$
5. $h = \frac{5}{2}$
6. $y = 7\sin\frac{\pi}{5}\left(x - \frac{5}{2}\right) + 6$
7. 

8. about 8.2 ft

Problem Solving

1. a. 2.5
   b. 12
   c. 2 units to the right
   d. 4 units up
   e. 6.5 m at 2:00 A.M.
   f. 1.5 m at 8:00 A.M.
2. 

3. 2
4. a. $d(t) = 4\cos\left(\frac{\pi}{6}\right)(t - 2) + 2.5$

   b. Possible answer: The maximum depth of the water would still be 6.5 m, but the minimum depth would be –1.5 m.