



Georgia Standards of Excellence Course Curriculum Overview

Mathematics

GSE Foundations of Algebra



Richard Woods, Georgia's School Superintendent
"Educating Georgia's Future"

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Foundations of Algebra Revision Summary

The Foundations of Algebra course has been revised based on feedback from teachers across the state. The following are changes made during the current revision cycle:

- Each module assessment has been revised to address alignment to module content, reading demand within the questions, and accessibility to the assessments by Foundations of Algebra teachers.
- All module assessments as well as the pre- and posttest for the course will now be available in GOFAR at the teacher level along with a more robust teacher’s edition featuring commentary along with the assessment items.
- All modules now contain “Quick Checks” that will provide information on mastery of the content at pivotal points in the module (or prerequisite skills they will need to be successful). Both teacher and student versions of the “Quick Checks” will be accessible within the module.
- A “Materials List” has been included in each module. The list provides teachers with materials that are needed for each lesson in that module.
- To draw attention to changes that have been made within the modules, corrections and additions to the content will be featured in **GREEN** font.
- A complete professional learning series with episodes devoted to the “big ideas” of each module and strategies for effective use of manipulatives will be featured on the Math Resources and Professional Learning page at <https://www.gadoe.org/Curriculum-Instruction-and-Assessment/Curriculum-and-Instruction/Pages/Mathematics.aspx>.
- **Additional support such as Module Analysis Tables may be found on the Foundations of Algebra page on the High School Math Wiki at <http://ccgpsmathematics9-10.wikispaces.com/Foundations+of+Algebra>.** This Module Analysis Table is NOT designed to be followed as a “to do list” but merely as ideas based on feedback from teachers of the course and professional learning that has been provided within school systems across Georgia.

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The Comprehensive Course Overview is designed to give teachers an understanding of the development and structure of Foundations of Algebra as well as give guidance in instructional practices. According to one of the module writers for the course, “Foundations of Algebra teachers have the unique opportunity to set a strong and well-appointed stage for future success in mathematics. You have the chance to tune into students’ math misunderstandings and begin to build a stronger foundation for future high school courses. Students must feel safe to ask questions....safe to make mistakes....safe to learn from each other...safe to question their own thinking....safe to question your thinking and explanations.”

Foundations of Algebra will provide many opportunities to revisit and expand the understanding of foundational algebra concepts, will employ diagnostic means to offer focused interventions, and will incorporate varied instructional strategies to prepare students for required high school courses. The course will emphasize both algebra and numeracy in a variety of contexts including number sense, proportional reasoning, quantitative reasoning with functions, and solving equations and inequalities.

Setting the Atmosphere for Success

“There is a huge elephant standing in most math classrooms, it is the idea that only some students can do well in mathematics. Students believe it; parents believe and teachers believe it. The myth that mathematics is a gift that some students have and some do not, is one of the most damaging ideas that pervades education in the US and that stands in the way of students’ mathematics achievement.” (Boaler, Jo. “Unlocking Children’s Mathematics Potential: 5 Research Results to Transform Mathematics Learning” youcubed at Stanford University. Web 10 May 2015.)

Some students believe that their ability to learn mathematics is a fixed trait, meaning either they are good at mathematics or not. This way of thinking is referred to as a **fixed mindset**. Other students believe that their ability to learn mathematics can develop or grow through effort and education, meaning the more they do and learn mathematics the better they will become. This way of thinking is referred to as a **growth mindset**.

In the fixed mindset, students are concerned about how they will be viewed, smart or not smart. These students do not recover well from setbacks or making mistakes and tend to “give up” or quit. In the growth mindset, students care about learning and work hard to correct and learn from their mistakes and look at these obstacles as challenges.

The manner in which students are praised greatly affects the type of mindset a student may exhibit. Praise for intelligence tends to put students in a fixed mindset, such as “You have it!” or “You are really good at mathematics”. In contrast, praise for effort tends to put students in a growth mindset, such as “You must have worked hard to get that answer.” or “You are developing mathematics skills because you are working hard”. Developing a growth mindset

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produces motivation, confidence and resilience that will lead to higher achievement. (Dweck, Carol. Mindset: The New Psychology of Success. Ballantine Books: 2007.)

“Educators cannot hand students confidence on a silver platter by praising their intelligence. Instead, we can help them gain the tools they need to maintain their confidence in learning by keeping them focused on the process of achievement.” (Dweck, Carol S. “The Perils and Promises of Praise.” *ASCD. Educational Leadership*. October 2007. Web 10 May 2015.)

Teachers know that the business of coming to know students as learners is simply too important to leave to chance and that the peril of not undertaking this inquiry is not reaching a learner at all. Research suggests that this benefit may improve a student’s academic performance. Surveying students’ interests in the beginning of a year will give teachers direction in planning activities that will “get students on board”. Several interest surveys are available and two examples can be located through the following websites:

https://www.scholastic.com/teachers/article/collateral_resources/pdf/student_survey.pdf

<http://www.union.k12.sc.us/ems/Teachers-Forms--Student%20Interest%20Survey.htm>

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Resources for Instruction

Along with the suggested web links below, Foundations of Algebra teachers have been provided with the following to use as the main resources for the course:

- Comprehensive Standards with Curriculum Map
- Five Modules which include scaffolded instructional lessons with **interventions** and formative assessments; the interventions table for each module is included in this overview after the Curriculum Map.
- “Quick Checks” that will provide information on mastery of the content at pivotal points in the module (or prerequisite skills they will need to be successful).
- Pre/Post assessments per module, housed on the Georgia Online Formative Assessment Resource (GOFAR).
Test anxiety can be addressed through such documents as <http://www.mathematicsgoodies.com/articles/how-to-reduce-mathematics-test-anxiety.html>
- Pre/post Course assessment, housed on GOFAR for district access
Test taking strategies can be very helpful for students. Two particular sites to find test taking strategies are: <http://www.regentsprep.org/regents/mathematics/geometry/StudyTips.htm>, and, from the College Board, <https://professionals.collegeboard.com/testing/sat-reasoning/prep/approach>.
- Assessment Commentaries, giving details on assessment answers and distractors
- Assessment Guides, which correlate the assessment items to the standards and provide Depth of Knowledge levels (DOK) for each item.
DOK mathematics’ examples can be found at: http://education.ky.gov/curriculum/docs/documents/cca_dok_support_808_.pdf
- The Individual Knowledge Assessment of Number, IKAN, with an instruction manual posted at: <http://www.gadoe.org/Curriculum-Instruction-and-Assessment/Curriculum-and-Instruction/Pages/.aspx>
- Monitoring Guidance Document with suggestions for schools and districts on monitoring student success <http://www.gadoe.org/Curriculum-Instruction-and-Assessment/Curriculum-and-Instruction/Pages/Mathematics.aspx>
- Georgia Public Broadcast video for administrators and counselors found at: <http://www.gadoe.org/Curriculum-Instruction-and-Assessment/Curriculum-and-Instruction/Pages/.aspx>

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Foundations of Algebra Curriculum Map

Georgia Standards of Excellence Foundations of Algebra Curriculum Map				
1st Semester			2nd Semester	
Module 1	Module 2	Module 3	Module 4	Module 5
Number Sense and Quantity	Arithmetic to Algebra	Proportional Reasoning	Equations and Inequalities	Quantitative Reasoning with Functions
MFANSQ1 MFANSQ2 MFANSQ3 MFANSQ4	MFAAA1 MFAAA2	MFAPR1 MFAPR2 MFAPR3	MFAEI1 MFAEI2 MFAEI3 MFAEI4	MFAQR1 MFAQR2 MFAQR3
All units will include the Mathematical Practices and indicate skills to maintain.				

NOTE: Mathematical standards are interwoven and should be addressed throughout the year in as many different modules and tasks as possible in order to stress the natural connections that exist among mathematical topics.

Foundations of Algebra Key:

NSQ = Number Sense and Quantity

AA = Arithmetic to Algebra

PR = Proportional Reasoning

EI = Equations and Inequalities

QR = Quantitative Reasoning with Functions

Table of Interventions by Module

Module 1 – Number Sense and Quantity

<u>Lesson Name</u>	<u>Name Of Intervention</u>	<u>Snapshot of summary or student I can statement ...</u>	<u>Book, Page Or link</u>
Building Number Sense Activities	Addition & Subtraction Pick-n-Mix	Use a range of additive and simple multiplicative strategies with whole numbers, fractions, decimals, and percentages.	Addition & Subtraction Pick-n-Mix
Fact Families	Bowl a Fact	Recall addition and subtraction facts to 20. Recall the multiplication and division facts for the multiples of 2, 3, 5, and 10. Recall multiplication to 10 x 10, and the corresponding division facts.	Bowl a Fact
Is It Reasonable?	Checking Addition and Subtraction by Estimation	Solve addition and subtraction problems by using place value	Checking Addition and Subtraction by Estimation Material Master 8-1
Birthday Cake	Chocolate Chip Cheesecake	Practice multiplying whole numbers by fractions	Chocolate Chip Cheesecake
Fraction Clues	Fractions in a Whole	Find unit fractions of sets using addition facts	Fractions in a Whole
	Hungry Birds	Find unit fractions of sets using addition facts	Hungry Birds
	Fraction Strategies: Wafers	Find unit fractions of sets using addition facts	Fraction Strategies: Wafers

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Multiplying Fractions	Multiplying Fractions	Work through some word problems to help increase fluency of multiplying fractions	Multiplying Fractions
Representing Powers of Ten Using Base Ten Blocks	Powers of Powers	Use these activities to help your students develop knowledge of place value and powers of 10 to support multiplicative thinking	Powers of Powers
Multiplying By Powers of Ten	Powers of Powers	Use these activities to help your students develop knowledge of place value and powers of 10 to support multiplicative thinking	Powers of Powers
Pattern-R-Us	Powers of Powers	Use these activities to help your students develop knowledge of place value and powers of 10 to support multiplicative thinking	Powers of Powers
Comparing Decimals	Arrow Cards	Compare Decimals using decimal arrow cards and an understanding of place value.	Arrow Cards Material Master
Are These Equivalent?	Bead Strings	Find equivalents for decimals and fractions.	Bead Strings
Integers on the Number Line	Bonus and Penalties	Allow students to build fluency and automatize their use and operations with integers.	Bonus and Penalties
Deep Freeze	Integer Quick Draw	Allow students to build fluency and automatize their use & operations with integers.	Integer Quick Draw
Multiplying Rational Numbers	Sign of the Times	Find a pattern in the multiplication facts of signed numbers.	Sign of the Times
Rational or Irrational	Recurring and Termination Decimals	Solve problems by finding the prime factors of numbers.	Recurring and Terminating Decimals

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Decimal Approximation of Roots	Tiling Teasers	Solve problems involving square roots	Tiling Teasers
Debits and Credits	Close to Zero	Allow students to build fluency and automatize their use and operations with integers.	Close to Zero

Module 2 – Arithmetic to Algebra

<u>Lesson Name</u>	<u>Name Of Intervention</u>	<u>Snapshot of summary or student I can statement ...</u>	<u>Book, Page Or link</u>
Olympic Cola Display	Animal Arrays	I am learning to find other ways to solve repeated addition problems.	Book 6 Page 15
	Array Game	The game allows students to practice their multiplication skills, and reinforces the ‘array’ concept of multiplication	Array Game
Distributing and Factoring Using Area	Multiplication Smorgasbord	I am learning to solve multiplication problems using a variety of mental strategies	Book 6 Page 56
	Smiley Hundred	In this activity, students are encouraged to solve multiplication problems by deriving from known facts, looking for groupings and skip counting. Students are encouraged to explain and share their thinking.	Smiley Hundred

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Conjectures About Properties	A Study of Number Properties	The purpose is to develop the students' deeper understanding of the way numbers behave, to enable them to use everyday language to make a general statement about these behaviors, and to understand the symbolic representation of these 'properties' of numbers and operations.	<u>A Study of Number Properties</u>
Translating Math	Displaying Postcards	Apply algebra to the solution of a problem Devise and use problem solving strategies to explore situations mathematically	<u>Displaying Postcards</u>
Exploring Expressions	Body Measurements	Design and use models to solve measuring problems in practical contexts.	<u>Body Measurements</u>
Squares, Area, Cubes, Volume, Connected?	Square and Cube Roots	Calculate square and cube roots. Understand that squaring is the inverse of square rooting, and cubing is the inverse of cube rooting.	<u>Square and Cube Roots</u>
What's the Hype about Pythagoras?	Gougu Rule or Pythagoras' Theorem	Find lengths of objects using Pythagoras' theorem.	<u>Gougu Rule or Pythagoras' Theorem</u>
	Pythagoras Power	Explore Pythagoras' Theorem.	<u>Pythagoras Power</u>

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Module 3 – Proportional Reasoning

<u>Lesson Name</u>	<u>Name Of Intervention</u>	<u>Snapshot of summary or student I can statement ...</u>	<u>Book, Page Or link</u>
Equivalent Fractions	Equivalent Fractions	The purpose of this activity is to help your child to practice finding equivalent fractions for numbers up to 100.	Equivalent Fractions
	Addition, subtraction and equivalent fractions	The purpose of this series of lessons is to develop understanding of equivalent fractions and the operations of addition and subtraction with fractions.	Addition, subtraction and equivalent fractions
Snack Mix	Mixing Colors	Solve problems involving ratios	Mixing Colors
What is a Unit Rate?	Breaking Records	Hands on activities where students can determine unit rate.	Unit Rate
Proportional Relationships	Enough Rice	Solve problems involving simple linear proportions	Enough Rice?
Orange Fizz Experiment	Fruit Proportions	Comparing proportions	Fruit Proportions
Which Is The Better Deal?	Percentages Resource Page	A series of 5 activities to help develop automaticity with percentages.	Percentages Resource Page
25% Sale	Percentages Resource Page	A series of 5 activities to help develop automaticity with percentages.	Percentages Resource Page

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Which Bed, Bath, and Beyond Coupon Should You Use?	Percentages Resource Page	A series of 5 activities to help develop automaticity with percentages.	Percentages Resource Page
What's My Line?	Rates of Changes	In this activity students solve problems involving unit rates	Rate of Changes
Nana's Chocolate Milk	Ratio	In this activity, students look at different mixtures.	Mixing Colors

Module 4 – Equations and Inequalities

<u>Lesson Name</u>	<u>Name Of Intervention</u>	<u>Snapshot of summary or student I can statement ...</u>	<u>Book, Page Or link</u>
The Variable Machine	Balancing Acts	The focus here is involving students in solving problems that can be modeled with algebraic equations or expressions. Students are required to describe patterns and relationships using letters to represent variables.	Balancing Acts
	Unknowns and Variables: Solving One Step Equations	The focus of learning is to develop an understanding of the symbols that we use to express our mathematical ideas and to communicate these ideas to others.	Unknowns & Variables: Solving One Step Equations
Set it Up	Holistic Algebra	The focus of learning is to relate tables, graphs, and equations to linear relationships found in number and spatial patterns.	Holistic Algebra

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Solving Equations using Bar Diagrams	Unknowns & Variables: Solving One Step Equations	Form and solve simple linear equations.	<u>Unknowns & Variables: Solving One Step Equations</u>
Deconstructing Word Problems	Writing in Words and Symbols	Use symbolic notation to record simple word phrases and explain what symbol notation says should be done to the numbers.	<u>Writing in Words and Symbols</u>
T.V. Time and Video Games	Multiplication & Division Symbols, Expressions & Relationships	Develop understanding of symbols for, and operations of multiplication and division, of their inverse relationship, and of how to use these operations in problem solving situations. (Sessions 1 and 2)	<u>Multiplication & Division Symbols, Expressions & Relationships</u>
Don't Sink My Battleship!	Graphic Detail	Connect members of sequential patterns with their ordinal position and use tables, graphs, and diagrams to find relationships between successive elements of number and spatial patterns	<u>Graphic Detail</u>
	Linear Graphs and Patterns	Relate tables, graphs, and equations	<u>Linear Graphs and Patterns</u>
Literal Equations	Magic Squares	Form and solve simple linear equations. Use graphs, tables, and rules, to describe linear relationships found in number and spatial patterns	<u>Magic Squares</u>
Free Throw Percentages	Weighing Time	Solve systems of equations	<u>Weighing Time</u>
Stacking Cups	Weighing Time	Solve systems of equations	<u>Weighing Time</u>
Planning a Party	Weighing Time	Solve systems of equations	<u>Weighing Time</u>

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Field Day	Weighing Time	Solve systems of equations	Weighing Time
Module 5 – Functions			
<u>Lesson Name</u>	<u>Name Of Intervention</u>	<u>Snapshot of summary or student I can statement ...</u>	<u>Book, Page Or link</u>
Reviewing Rate of Change	Rates of Change	Solve problems involving rates.	Rates of Change

Standards for Mathematical Practice (Grades 5 – high school)

The Standards for Mathematical Practice describe varieties of expertise that educators at all levels should seek to develop in their students. These practices rest on important “processes and proficiencies” with longstanding importance in education. The first of these are the National Council of Teachers of Mathematics (NCTM) process standards of problem solving, reasoning and proof, communication, representation, and connections. (*Principles and Standards for School Mathematics*. NCTM: 2000.) The second are the strands of mathematical proficiency specified in the National Research Council’s report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently, and appropriately), and productive disposition (habitual inclination to see as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy) (National Academies Press, 2001.)

Students are expected to:

1. Make sense of problems and persevere in solving them.

Students begin in elementary school to solve problems by applying their understanding of operations with whole numbers, decimals, and fractions including mixed numbers. Students seek the meaning of a problem and look for efficient ways to represent and solve it. In middle school, students solve real world problems through the application of algebraic and geometric concepts.

High school students start to examine problems by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. They check their answers to problems using different methods and continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2. Reason abstractly and quantitatively.

Earlier grade students should recognize that a number represents a specific quantity. They connect quantities to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities. They extend this understanding from whole numbers to their work with fractions and decimals. Students write simple expressions that record calculations with numbers and represent or round numbers using place value concepts.

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In middle school, students represent a wide variety of real world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities. They examine patterns in data and assess the degree of linearity of functions. Students contextualize to understand the meaning of the number or variable as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations.

High school students seek to make sense of quantities and their relationships in problem situations. They abstract a given situation and represent it symbolically, manipulate the representing symbols, and pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Students use quantitative reasoning to create coherent representations of the problem at hand; consider the units involved; attend to the meaning of quantities, not just how to compute them; and know and flexibly use different properties of operations and objects.

3. Construct viable arguments and critique the reasoning of others.

In earlier grades, students may construct arguments using concrete referents, such as objects, pictures, and drawings. They explain calculations based upon models and properties of operations and rules that generate patterns. They demonstrate and explain the relationship between volume and multiplication.

In middle school, students construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, and graphs, tables, and other data displays (e.g., box plots, dot plots, histograms). They further refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students. The students pose questions like “How did you get that?”, “Why is that true?”, and “Does that always work?” They explain their thinking to others and respond to others’ thinking.

High school students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. High school students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and — if there is a flaw in an argument — explain what it is. High school students learn to determine domains to which an argument applies, listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. Model with Mathematics.

Students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, making a chart, list, or graph, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed. Elementary students should evaluate their results in the context of the situation and whether the results make sense. They also evaluate the utility of models to determine which models are most useful and efficient to solve problems.

In middle school, students model problem situations with symbols, graphs, tables, and context. Students form expressions, equations, or inequalities from real world contexts and connect symbolic and graphical representations. Students solve systems of linear equations and compare properties of functions provided in different forms. Students use scatterplots to represent data and describe associations between variables. Students need many opportunities to connect and explain the connections between the different representations. They should be able to use all of these representations as appropriate to a problem context.

High school students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. High school students make assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically.

Elementary students consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, they may use unit cubes to fill a rectangular prism and then use a ruler to measure the dimensions. They use graph paper to accurately create graphs and solve problems or make predictions from real world data.

Students in middle school may translate a set of data given in tabular form to a graphical representation to compare it to another data set. Students might draw pictures, use applets, or write equations to show the relationships between the angles created by a transversal.

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High school students' tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. High school students should be sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. They are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. Attend to precision.

Students in earlier grades continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to expressions, fractions, geometric figures, and coordinate grids. They are careful about specifying units of measure and state the meaning of the symbols they choose. For instance, when figuring out the volume of a rectangular prism they record their answers in cubic units. Students in middle school use appropriate terminology when referring to the number system, functions, geometric figures, and data displays.

High school students try to communicate precisely to others by using clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, and express numerical answers with a degree of precision appropriate for the problem context. By the time they reach high school, they have learned to examine claims and make explicit use of definitions.

7. Look for and make use of structure.

In elementary grades, students look closely to discover a pattern or structure. For instance, students use properties of operations as strategies to add, subtract, multiply, and divide with whole numbers, fractions, and decimals. They examine numerical patterns and relate them to a rule or a graphical representation.

Students in middle school routinely seek patterns or structures to model and solve problems. Students apply properties to generate equivalent expressions and solve equations. Students examine patterns in tables and graphs to generate equations and describe relationships. Additionally, students experimentally verify the effects of transformations and describe them in terms of congruence and similarity.

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By high school, students look closely to discern a pattern or structure. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y . High school students use these patterns to create equivalent expressions, factor and solve equations, compose functions, and transform figures.

8. Look for and express regularity in repeated reasoning.

Students in elementary grades use repeated reasoning to understand algorithms and make generalizations about patterns. Students connect place value and their prior work with operations to understand algorithms, to fluently multiply multi-digit numbers, and to perform all operations with decimals to hundredths. Students explore operations with fractions with visual models and begin to formulate generalizations.

Middle school students use repeated reasoning to understand algorithms and make generalizations about patterns. Students use iterative processes to determine more precise rational approximations for irrational numbers. During multiple opportunities to solve and model problems, they notice that the slope of a line and rate of change are the same value. Students flexibly make connections between covariance, rates, and representations showing the relationships between quantities.

High school students notice if calculations are repeated and look both for general methods and for shortcuts. As they work to solve a problem, derive formulas, or make generalizations, high school students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

More information of the Standards for Mathematical Practice may be found on the [Inside Mathematics](#) website.

Connecting the Standards for Mathematical Practice to the Content Standards

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly should engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in instruction.

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The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word “understand” are often especially good opportunities to connect the practices to the content. Students who are missing the understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, an absence of understanding effectively prevents a student from engaging in the mathematical practices.

In this respect, those content standards which set an expectation of understanding are potential “points of intersection” between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward the central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics. See [Inside Mathematics](#) for more resources.

Content Standards

The content standards for Foundations of Algebra are an amalgamation of mathematical standards addressed in grades 3 through high school.

After each Foundations of Algebra standard there is a list of reference standards in blue. These reference standards refer to the standards used to form those for Foundations of Algebra.

Module 1 - Number Sense and Quantity

Students will compare different representations of numbers (i.e. fractions, decimals, radicals, etc.) and perform basic operations using these different representations.

Numbers, Number Sense, and Number Systems: During the years from kindergarten to eighth grade, students must repeatedly extend their conception of number. At first, “number” means “counting number”: 1, 2, 3... Soon after that, 0 is used to represent “none”, and the whole numbers are formed by the counting numbers together with zero. Number sense essentially refers to a student’s ability to think fluidly and flexibly about numbers. To be successful in mathematics students need to have a sense that numbers are meaningful, i.e. what numbers mean and how they are related to one another. Additionally, they need to understand symbolic representations, use and understand numbers in real world contexts, and be able to perform mental math. Students who lack strong number sense do not have the foundation needed for simple arithmetic, much less more complex math. Building strong number sense is important because it promotes a sense of confidence in “making friends with numbers” (Carlyle and Mercado 2012). The next extension is fractions. At first, fractions are barely numbers and tied strongly to pictorial representations. Yet by the time students understand division of fractions, they have a strong concept of fractions as numbers and have connected them, via their decimal representations, with the base-ten system used to represent the whole numbers. During middle school, fractions are augmented by negative fractions to form the rational numbers. In Grade 8, students extend this system once more, augmenting the rational numbers with the irrational numbers to form the real numbers. In high school, students will be exposed to yet another extension of number, when the real numbers are augmented by the imaginary numbers to form the complex numbers. With each extension of number, the meanings of addition, subtraction, multiplication, and division are extended. In each new number system — integers, rational numbers, real numbers, and complex numbers — the four operations stay the same in two important ways: They have the commutative, associative, and distributive properties, and their new meanings are consistent with their previous meanings. Extending the properties of whole-number exponents leads to new and productive notation. For example, properties of whole-number exponents suggest that $(5^{1/3})^3$ should be $5^{(1/3)3} = 5^1 = 5$.

Quantities: In real world problems, the answers are usually not numbers but quantities: numbers with units, which involves measurement. In their work in measurement through eighth grade, students primarily measure commonly used attributes such as length, area, and volume. In high school, students encounter a wider variety of units in modeling, e.g., acceleration, currency conversions, derived quantities such as person-hours and heating degree days, social science rates such as per-capita income, and rates in everyday life such as points scored per game or batting averages. They also encounter novel situations in which they themselves must conceive the attributes of interest. For example, to find a good measure of overall highway safety, they might propose measures such as fatalities per year, fatalities per year per driver, or fatalities per vehicle-mile traveled. Such a conceptual process is sometimes called quantification. Quantification is important for science, as when surface area suddenly “stands out” as an

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important variable in evaporation. Quantification is also important for companies, which must conceptualize relevant attributes and create or choose suitable measures for them.

“Flip Books”: The “Flip Books”, linked below on each middle and high school reference standard, were developed by the Kansas Association of Teachers of Mathematics (KATM) and are a compilation of research, “unpacked” standards from many states, instructional strategies, and examples for each standard at each grade level. The intent is to show the connections to the Standards of Mathematical Practices for the content standards and to get detailed information at each level. The [Middle School Flip Books](#) and [High School Flip Book](#) are interactive documents arranged by the content domains listed on the following pages. The links on each domain and standard will take you to specific information on that standard/domain within the Flip Book.

MFANSQ1. Students will analyze number relationships.

- Solve multi-step real world problems, analyzing the relationships between all four operations. *For example, understand division as an unknown-factor problem in order to solve problems. Knowing that $50 \times 40 = 2000$ helps students determine how many boxes of cupcakes they will need in order to ship 2000 cupcakes in boxes that hold 40 cupcakes each.* ([MGSE3.OA.6](#), [MGSE4.OA.3](#))
- Understand a fraction a/b as a multiple of $1/b$. ([MGSE4.NF.4](#))
- Explain patterns in the placement of decimal points when multiplying or dividing by powers of ten. ([MGSE5.NBT.2](#))
- Compare fractions and decimals to the thousandths place. *For fractions, use strategies other than cross multiplication. For example, locating the fractions on a number line or using benchmark fractions to reason about relative size. For decimals, use place value.* ([MGSE4.NF.2](#);[MGSE5.NBT.3,4](#))

MFANSQ2. Students will conceptualize positive and negative numbers (including decimals and fractions).

- Explain the meaning of zero. ([MGSE6.NS.5](#))
- Represent numbers on a number line. ([MGSE6.NS.5,6](#))
- Explain meanings of real numbers in a real world context. ([MGSE6.NS.5](#))

MFANSQ3. Students will recognize that there are numbers that are not rational, and approximate them with rational numbers.

- Find an estimated decimal expansion of an irrational number locating the approximations on a number line. *For example, for $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue this pattern in order to obtain better approximations.* ([MGSE8.NS.1,2](#))

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- b. Explain the results of adding and multiplying with rational and irrational numbers. ([MGSE9-12.N.RN.3](#))

MFANSQ4. Students will apply and extend previous understanding of addition, subtraction, multiplication, and division.

- a. Find sums, differences, products, and quotients of multi-digit decimals using strategies based on place value, the properties of operations, and/or relationships between operations. ([MGSE5.NBT.7](#); [MGSE6.NS.3](#))
- b. Find sums, differences, products, and quotients of all forms of rational numbers, stressing the conceptual understanding of these operations. ([MGSE7.NS.1.2](#))
- c. Interpret and solve contextual problems involving division of fractions by fractions. *For example, how many $\frac{3}{4}$ -cup servings are in $\frac{2}{3}$ of a cup of yogurt?* ([MGSE6.NS.1](#))
- d. Illustrate and explain calculations using models and line diagrams. ([MGSE7.NS.1.2](#))
- e. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using estimation strategies and graphing technology. ([MGSE7.NS.3](#), [MGSE7.EE.3](#), [MGSE9-12.N.Q.3](#))

Elementary Reference Standards

These reference standards refer to the standards used to form the standards for the Foundations of Algebra course. Below, you will find the elementary reference standards with instructional strategies and common misconceptions.

MGSE.3.OA.6. Understand division as an unknown-factor problem. *For example, find $32 \div 8$ by finding the number that makes 32 when multiplied by 8.*

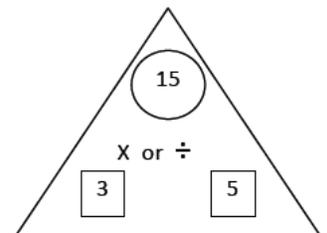
Since multiplication and division are inverse operations, students are expected to solve problems and explain their processes of solving division problems that can also be represented as unknown factor multiplication problems.

Example: A student knows that $2 \times 9 = 18$. How can they use that fact to determine the answer to the following question: 18 people are divided into pairs in a P.E. class? How many pairs are there? Write a division equation and explain your reasoning.

Multiplication and division are inverse operations and that understanding can be used to find the unknown. Fact family triangles demonstrate the inverse operations of multiplication and division by showing the two factors and how those factors relate to the product and/or quotient. Examples:

$$3 \times 5 = 15 \qquad 5 \times 3 = 15$$

$$15 \div 3 = 5 \qquad 15 \div 5 = 3$$



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MGSE.4.OA.3. Solve multistep word problems with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a symbol or letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding

The focus in this standard is to have students use and discuss various strategies. The reference is to estimation strategies, including using compatible numbers (numbers that sum to 10 or 100) or rounding. Problems should be structured so that all acceptable estimation strategies will arrive at a reasonable answer. Students need many opportunities solving multistep story problems using all four operations.

Example 1:

On a vacation, your family travels 267 miles on the first day, 194 miles on the second day and 34 miles on the third day. How many miles did they travel total?

Some typical estimation strategies for this problem are shown below:

<p>Student 1:</p> <p>I first thought about 267 and 34. I noticed that their sum is about 300. Then I knew that 194 is close to 200. When I put 300 and 200 together, I get 500.</p>	<p>Student 2:</p> <p>I first thought about 194. It is really close to 200. I also have 2 hundreds in 267. That gives me a total of 4 hundreds. Then I have 67 in 267 and the 34. When I put 67 and 34 together that is really close to 100. When I add that hundred to the 4 hundreds that I already had, I end up with 500.</p>	<p>Student 3:</p> <p>I rounded 267 to 300. I rounded 194 to 200. I rounded 34 to 30. When I added 300, 200, and 30, I know my answer will be about 530.</p>
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The assessment of estimation strategies should have a reasonable answer (500 or 530), or a range (between 500 and 550). Problems will be structured so that all acceptable estimation strategies will arrive at a reasonable answer.

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Example 2:

Your class is collecting bottled water for a service project. The goal is to collect 300 bottles of water. On the first day, Max brings in 3 containers with 6 bottles in each container. Sarah wheels in 6 containers with 6 bottles in each container. About how many bottles of water still need to be collected?

Student 1:

First I multiplied 3 and 6 which equals 18. Then I multiplied 6 and 6 which is 36. I know 18 plus 36 is about 50. I'm trying to get to 300. 50 plus another 50 is 100. Then I need 2 more hundreds. So we still need 250 bottles.

Student 2:

First I multiplied 3 and 6 which equals 18. Then I multiplied 6 and 6 which is 36. I know 18 is about 20 and 36 is about 40. $40 + 20 = 60$. $300 - 60 = 240$, so we need about 240 more bottles.

This standard also references interpreting remainders. Remainders should be put into context for interpretation. Ways to address remainders:

- Remained as a left over
- Partitioned into fractions or decimals
- Discarded leaving only the whole number answer
- Increased the whole number answer up one
- Rounded to the nearest whole number for an approximate result

Example 1:

Write different word problems involving $44 \div 6 = ?$ where the answers are best represented as:

- Problem A: 7
- Problem B: 7 r 2
- Problem C: 8
- Problem D: 7 or 8
- Problem E: $7 \frac{2}{6}$

Possible solutions:

Problem A: 7.

Mary had 44 pencils. Six pencils fit into each of her pencil pouches. How many pouches did she fill? $44 \div 6 = p$; $p = 7 \text{ r } 2$. *Mary can fill 7 pouches completely.*

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Problem B: 7 r 2.

Mary had 44 pencils. Six pencils fit into each of her pencil pouches. How many pouches could she fill and how many pencils would she have left? $44 \div 6 = p$; $p = 7 \text{ r } 2$; *Mary can fill 7 pouches and have 2 left over.*

Problem C: 8.

Mary had 44 pencils. Six pencils fit into each of her pencil pouches. What would the fewest number of pouches she would need in order to hold all of her pencils? $44 \div 6 = p$; $p = 7 \text{ r } 2$; *Mary needs 8 pouches to hold all of the pencils.*

Problem D: 7 or 8.

Mary had 44 pencils. She divided them equally among her friends before giving one of the leftovers to each of her friends. How many pencils could her friends have received? $44 \div 6 = p$; $p = 7 \text{ r } 2$; *some of her friends received 7 pencils. Two friends received 8 pencils.*

Problem E: $7\frac{2}{6}$.

Mary had 44 pencils and put six pencils in each pouch. What fraction represents the number of pouches that Mary filled? $44 \div 6 = p$; $p = 7\frac{2}{6}$

Example 2:

There are 128 students going on a field trip. If each bus held 30 students, how many buses are needed? ($128 \div 30 = b$; $b = 4 \text{ R } 8$). *They will need 5 buses because 4 busses would not hold all of the students).*

Students need to realize in problems, such as the examples above, that an extra bus is needed for the 8 students that are left over. Estimation skills include identifying when estimation is appropriate, determining the level of accuracy needed, selecting the appropriate method of estimation, and verifying solutions or determining the reasonableness of situations using various estimation strategies. Estimation strategies include, but are not limited to the following:

- **Front-end estimation with adjusting** (Using the highest place value and estimating from the front end, making adjustments to the estimate by taking into account the remaining amounts)
- **Clustering around an average** (When the values are close together, an average value is selected and multiplied by the number of values to determine an estimate.)
- **Rounding and adjusting** (Students round down or round up and then adjust their estimate depending on how much the rounding affected the original values.)

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- **Using friendly or compatible numbers such as factors** (Students seek to fit numbers together; e.g., rounding to factors and grouping numbers together that have round sums like 100 or 1000.)
- **Using benchmark numbers that are easy to compute** (Students select close whole numbers for fractions or decimals to determine an estimate.)

MGSE.4.NF.2. Compare two fractions with different numerators and different denominators, e.g., by using visual fraction models, by creating common denominators or numerators, or by comparing to a benchmark fraction such as $\frac{1}{2}$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols $>$, $=$, or $<$, and justify the conclusions.

This standard asks students to compare fractions by creating visual fraction models or finding common denominators or numerators. **Students' experiences should focus on visual fraction models rather than algorithms.** When tested, models may or may not be included. Students should learn to draw fraction models to help them compare. Students must also recognize that they must consider the size of the whole when comparing fractions (i.e., $\frac{1}{2}$ and $\frac{1}{8}$ of two medium pizzas is very different from $\frac{1}{2}$ of one medium and $\frac{1}{8}$ of one large).

Example 1:

Use pattern blocks.

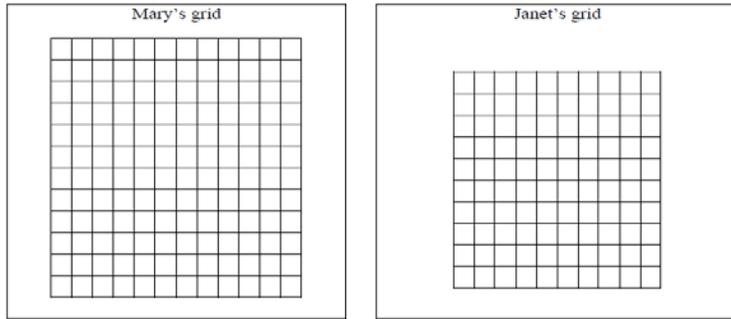
- If a red trapezoid is one whole, which block shows $\frac{1}{3}$?
- If the blue rhombus is $\frac{1}{3}$, which block shows one whole?
- If the red trapezoid is one whole, which block shows $\frac{2}{3}$?

Example 2:

Mary used a 12×12 grid to represent 1 and Janet used a 10×10 grid to represent 1. Each girl shaded grid squares to show $\frac{1}{4}$. How many grid squares did Mary shade? How many grid squares did Janet shade? Why did they need to shade different numbers of grid squares?

Possible solution: Mary shaded 36 grid squares; Janet shaded 25 grid squares. The total number of little squares is different in the two grids, so $\frac{1}{4}$ of each total number is different.

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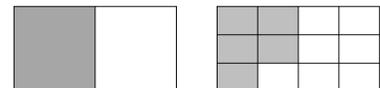


Example 3:

There are two cakes on the counter that are the same size. The first cake has $\frac{1}{2}$ left. The second cake has $\frac{5}{12}$ left. Which cake has more left?

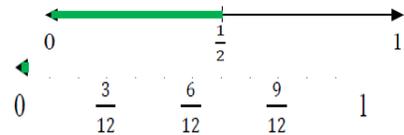
Student 1: *Area Model*

The first cake has more left over. The second cake has $\frac{5}{12}$ left which is smaller than $\frac{1}{2}$.



Student 2: *Number Line Model*

The first cake has more left over: $\frac{1}{2}$ is bigger than $\frac{5}{12}$

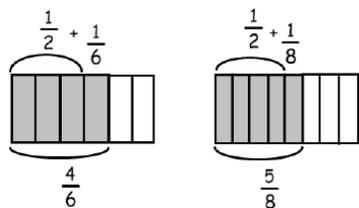


Student 3: *Verbal Explanation*

I know that $\frac{6}{12}$ equals $\frac{1}{2}$, and $\frac{5}{12}$ is less than $\frac{1}{2}$. Therefore, the second cake has less left over than the first cake. The first cake has more left over.

Example 4:

When using the benchmark of $\frac{1}{2}$ to compare $\frac{4}{6}$ and $\frac{5}{8}$, you could use diagrams such as these:



$\frac{4}{6}$ is $\frac{1}{6}$ larger than $\frac{1}{2}$, while $\frac{5}{8}$ is $\frac{1}{8}$ larger than $\frac{1}{2}$. Since $\frac{1}{6}$ is greater than $\frac{1}{8}$, $\frac{4}{6}$ is the greater fraction.

Common Misconceptions

Students think that when generating equivalent fractions they need to multiply or divide either the numerator or denominator by the same number rather than the numerator **and** denominator. For example, when making equivalent fractions for $\frac{5}{6}$, a student may multiply just the numerator by 2 resulting in $\frac{10}{6}$ instead of correctly multiplying by $\frac{2}{2}$ with the result $\frac{10}{12}$. **This misconception comes about because students do not understand that they need to use a fraction in the form of one**, such as $\frac{2}{2}$ to generate an equivalent fraction. **Reviewing the identity property with students reemphasizing what happens when we multiply by 1 is an essential component of instruction when addressing this misconception. Conversation centered around “what one (in disguise) could we use to create an equivalent fraction?”**

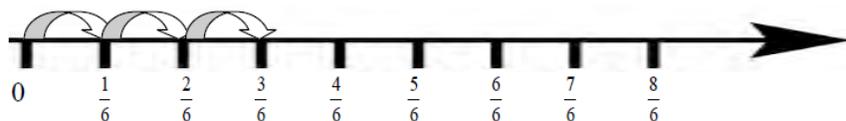
MGSE.4.NF.4. Apply and extend previous understandings of multiplication to multiply a fraction by a whole number e.g., by using a visual such as a number line or area model.

a. Understand a fraction a/b as a multiple of $1/b$. For example, use a visual fraction model to represent $\frac{5}{4}$ as the product $5 \times (\frac{1}{4})$, recording the conclusion by the equation $\frac{5}{4} = 5 \times (\frac{1}{4})$.

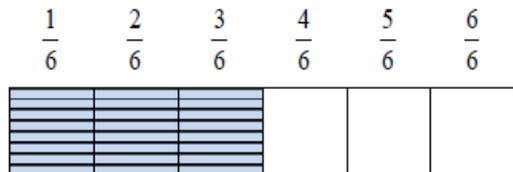
This standard builds on students’ work of adding fractions and extending that work into multiplication.

Example: $\frac{3}{6} = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = 3 \times \frac{1}{6}$

Number line:



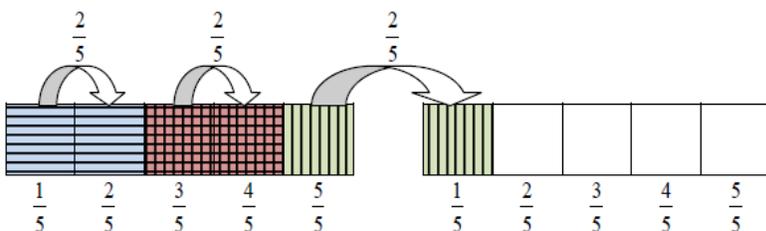
Area model:



b. Understand a multiple of a/b as a multiple of $1/b$, and use this understanding to multiply a fraction by a whole number. For example, use a visual fraction model to express $3 \times (\frac{2}{5})$ as $6 \times (\frac{1}{5})$, recognizing this product as $(\frac{6}{5})$. (In general, $n \times (a/b) = (n \times a)/b$.)

This standard extended the idea of multiplication as repeated addition. For example, $3 \times \frac{2}{5} = \frac{2}{5} + \frac{2}{5} + \frac{2}{5} = \frac{6}{5} = 6 \times \frac{1}{5}$.

Students are expected to use and create visual fraction models to multiply a whole number by a fraction.



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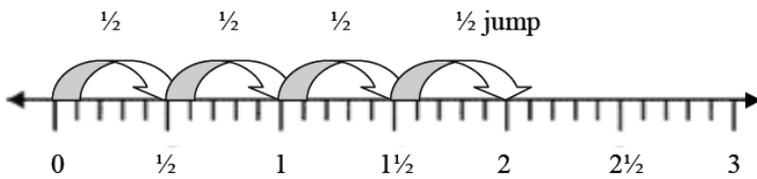
c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat $\frac{3}{8}$ of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*

This standard calls for students to use visual fraction models to solve word problems related to multiplying a whole number by a fraction.

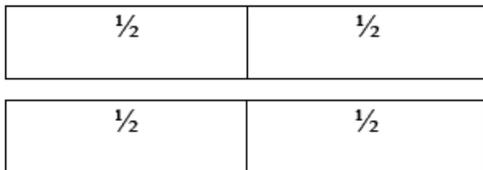
Example 1:

In a relay race, each runner runs $\frac{1}{2}$ of a lap. If there are 4 team members how long is the race?

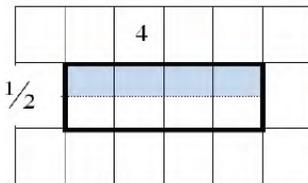
Student 1: Draws a number line showing 4 jumps of $\frac{1}{2}$:



Student 2: Draws an area model showing 4 pieces of $\frac{1}{2}$ joined together to equal 2:



Student 3: Draws an area model representing $4 \times \frac{1}{2}$ on a grid, dividing one row into $\frac{1}{2}$ to represent the multiplier:



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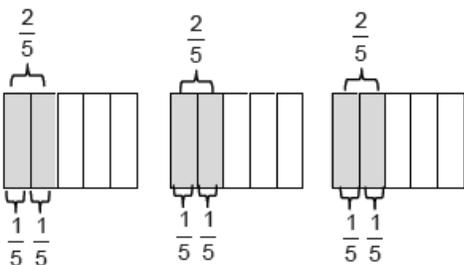
Example 2:

Heather bought 12 plums and ate $\frac{1}{3}$ of them. Paul bought 12 plums and ate $\frac{1}{4}$ of them. Which statement is true? Draw a model to explain your reasoning.

- Heather and Paul ate the same number of plums.
- Heather ate 4 plums and Paul ate 3 plums.
- Heather ate 3 plums and Paul ate 4 plums.
- Heather had 9 plums remaining.

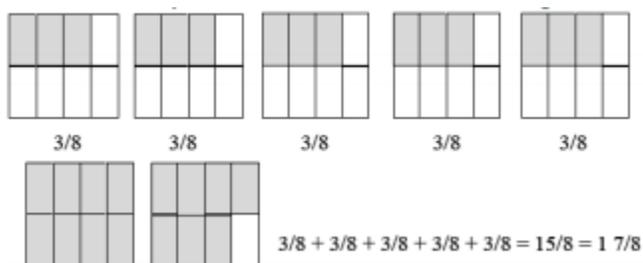
Example 3: Students need many opportunities to work with problems in context to understand the connections between models and corresponding equations. Contexts involving a whole number times a fraction lend themselves to modeling and examining patterns.

a. $3 \times \frac{2}{5} = 6 \times \frac{1}{5} = \frac{6}{5}$



- b. If each person at a party eats $\frac{3}{8}$ of a pound of roast beef, and there are 5 people at the party, how many pounds of roast beef are needed? Between what two whole numbers does your answer lie?

A student may build a fraction model to represent this problem:



Common Misconception

Students think that it does not matter which model to use when finding the sum or difference of fractions. They may represent one fraction with a rectangle and the other fraction with a circle. They need to know that the models need to represent the same whole.

MGSE.5.NBT.2 Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.

This standard includes multiplying by multiples of 10 and powers of 10, including 10^2 which is $10 \times 10 = 100$, and 10^3 which is $10 \times 10 \times 10 = 1,000$. Students should have experiences working with connecting the pattern of the number of zeros in the product when you multiply by powers of 10.

Example 1: $2.5 \times 10^3 = 2.5 \times (10 \times 10 \times 10) = 2.5 \times 1,000 = 2,500$

Students should reason that the exponent above the 10 indicates how many places the decimal point is moving (not just that the decimal point is moving but that you are multiplying or making the number 10 times greater three times) when you multiply by a power of 10. Since we are multiplying by a power of 10 the decimal point moves to the right.

Example 2: $350 \div 10^3$

$350 \div 10^3 = 350 \div 1,000$ (This extra representation should be added so that the decimal is just not “moving”) $= 0.350 = 0.35$

Example 3: $^{350}/_{10}$

$^{350}/_{10} = (350 \times ^{1}/_{10}) = ^{35}/_1 = 35$ The second step emphasizes that the “0” just does not disappear. This example shows that when we divide by powers of 10, the exponent above the 10 indicates how many places the decimal point is moving (how many times we are dividing by 10, the number becomes ten times smaller). Since we are dividing by powers of 10, the decimal point moves to the left. Students need to be provided with opportunities to explore this concept and come to this understanding; this **should not just be taught procedurally**.

MGSE.5.NBT.3. Read, write, and compare decimals to thousandths.

- a. **Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., $347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times (1/10) + 9 \times (1/100) + 2 \times (1/1000)$.**
- b. **Compare two decimals to thousandths based on meanings of the digits in each place, using $>$, $=$, and $<$ symbols to record the results of comparisons.**

This standard references the expanded form of decimals with fractions included. Students should build on their work from 4th grade, where they worked with both decimals and fractions interchangeably. Expanded form is included to build upon work in **MGSE.5.NBT.2** and to deepen students’ understanding of place value.

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Students will also build on the understanding they developed in fourth grade to read, write, and compare decimals to thousandths. They will connect to their prior experiences of using decimal notation for fractions in the addition of fractions with denominators of 10 and 100. **When dealing with tenths and hundredths, conversation about money (cents) can help students make connections to using decimals in real world contexts.**

Students will benefit by using concrete models and number lines to read, write, and compare decimals to the thousandths. Models may include base ten blocks, place value charts, grids, pictures, drawings, manipulatives, website virtual manipulatives etc. They will need to read decimals using fractional language and to write decimals in fractional form, as well as in expanded notation. This investigation leads to understanding equivalence of decimals ($0.8 = 0.80 = 0.800$).

Example:

Some equivalent forms of 0.72 are:

$$\begin{array}{ll} \frac{72}{100} & \frac{70}{100} + \frac{2}{100} \\ \frac{7}{10} + \frac{2}{100} & 0.720 \\ 7 \times (\frac{1}{10}) + 2 \times (\frac{1}{100}) & 7 \times (\frac{1}{10}) + 2 \times (\frac{1}{100}) + 0 \times (\frac{1}{1000}) \\ 0.70 + 0.02 & \frac{720}{1000} \end{array}$$

Students need to conceptually understand the size of decimal numbers and to relate them to common benchmarks such as 0, 0.5 (0.50 and 0.500), and 1. Comparing tenths to tenths, hundredths to hundredths, and thousandths to thousandths is simplified if students use their understanding of fractions to compare decimals.

Example:

Comparing 0.25 and 0.17, a student might think, “25 hundredths is more than 17 hundredths”. They may also think that it is 8 hundredths more. They may write this comparison as $0.25 > 0.17$ and recognize that $0.17 < 0.25$ is another way to express this comparison. **Additionally, connecting to money, “\$0.25 is more than \$0.17” or “\$0.17 is less than \$0.25”**

Comparing 0.207 to 0.26, a student might think, “Both numbers have 2 tenths, so I need to compare the hundredths. The second number has 6 hundredths and the first number has no hundredths so the second number must be larger. Another student might think while writing fractions, “I know that 0.207 is 207 thousandths (and may write $\frac{207}{1000}$). 0.26 is 26 hundredths (and may write $\frac{26}{100}$) but I can also think of it as 260 thousandths ($\frac{260}{1000}$). So, 260 thousandths is more than 207 thousandths.

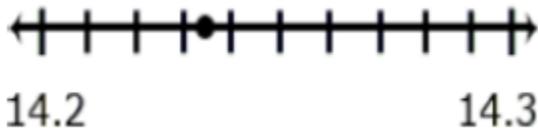
MGSE.5.NBT.4. Use place value understanding to round decimals up to the hundredths place

This standard refers to rounding. Students should go beyond simply applying an algorithm or procedure for rounding. The expectation is that students have a deep understanding of place value and number sense and can explain and reason about the answers they get when they round. Students should have numerous experiences using a number line to support their work with rounding.

Example:

Round 14.235 to the nearest tenth.

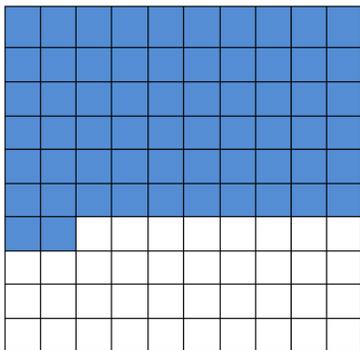
Students **must** recognize that the possible answer must be in tenths thus, it is either 14.2 or 14.3. They then identify that 14.235 is closer to 14.2 (14.20) than to 14.3 (14.30).



Students should use benchmark numbers to support this work. Benchmarks are convenient numbers for comparing and rounding numbers. 0, 0.5, 1, 1.5 are examples of benchmark numbers.

Example:

Which benchmark number is the best estimate of the shaded amount in the model below? Explain your thinking.



In this case 0.62 would be the number to be rounded to the nearest tenth. By seeing that the “extra blocks” are closest to $\frac{60}{100}$, the colored blocks show that 0.6 would be the closest value.

Common Misconceptions

A misconception that is directly related to the comparison of whole numbers and the comparison of decimals is the idea that the more digits a number contains means the greater the value of the number. With whole numbers, a 5-digit number is always greater than a 1-, 2-, 3-, or 4-digit number. However, with decimals, a number with one decimal place may be greater than a number with two or three decimal places. For example, 0.5 is greater than 0.12, 0.009 or 0.499. One method for comparing decimals is to rewrite all numbers so that they have the same number of digits to the right of the decimals point, such as 0.500, 0.120, 0.009 and 0.499. A second method is to use a place-value chart to place the numerals for comparison. **A third would be to think of the amount as money and have students' first compare the "cents" decimal positions (tenths and hundredths) to determine how much "change" they are looking at for each example.**

MGSE.5.NBT.7 Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

This standard also builds on work begun in 4th grade when students were introduced to decimals and asked to compare them. In 5th grade, students begin adding, subtracting, multiplying and dividing decimals. This work should focus on concrete models and pictorial representations, rather than relying solely on the algorithm. The use of symbolic notations involves having students record the answers to computations ($2.25 \times 3 = 6.75$), but this work should not be done without models or pictures. This standard requires that students explain their reasoning and how they use models, pictures, and strategies. Students are expected to extend their understanding of whole number models and strategies to decimal values. Before students are asked to give exact answers, they should estimate answers based on their understanding of operations and the value of the numbers.

Examples:

5.4 – 0.8

A student might estimate the answer to be a little more than 4.4 because a number less than 1 is being subtracted.

6 × 2.4

A student might estimate an answer between 12 and 18 since 6×2 is 12 and 6×3 is 18. Another student might give an estimate of a little less than 15 because s/he figures the answer to be very close, but smaller than $6 \times 2\frac{1}{2}$ and think of $2\frac{1}{2}$ groups of 6 as 12 (2 groups of 6) + $3(\frac{1}{2}$ of a group of 6).

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When adding or subtracting decimals, students should be able to explain that tenths are added or subtracted from tenths and hundredths are added or subtracted from hundredths. So, students will need to communicate that when adding or subtracting in a vertical format (numbers beneath each other), it is important that digits with the same place value are written in the same column. This understanding can be reinforced by linking the decimal addition and subtraction process to addition and subtraction of fractions. Adding and subtracting fractions with like denominators of 10 and 100 is a standard in fourth grade.

Common Misconceptions

Students might compute the sum or difference of decimals by lining up the right-hand digits as they would whole number. For example, in computing the sum of $15.34 + 12.9$, students will write the problem in this manner:

15.34

+ 12.9

16.63

To help students add and subtract decimals correctly, have them first estimate the sum or difference. Providing students with a decimal-place value chart will enable them to place the digits in the proper place.

Example 1: $4 - 0.3$

3 tenths subtracted from 4 wholes. One of the wholes must be divided into tenths.



The solution is $3 + \frac{7}{10}$ or 3.7.

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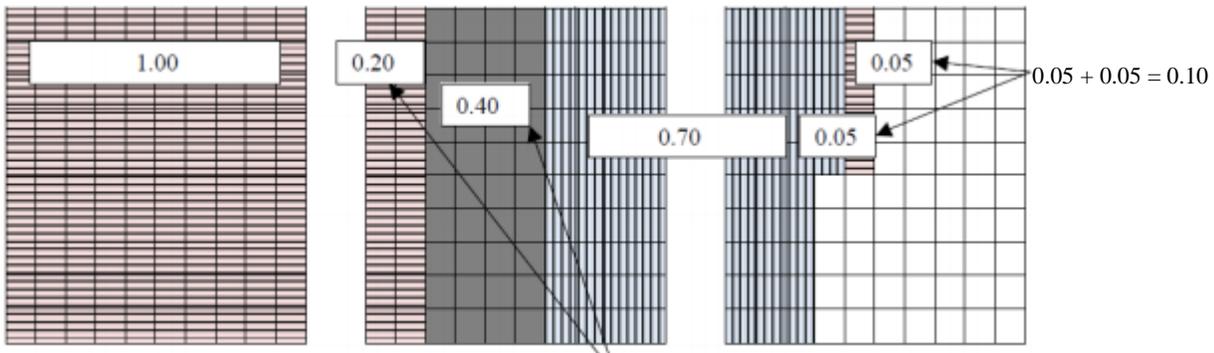
Example 2:

A recipe for a cake requires 1.25 cups of milk, 0.40 cups of oil, and 0.75 cups of water. How much liquid is in the mixing bowl?

Student 1: $1.25 + 0.40 + 0.75$

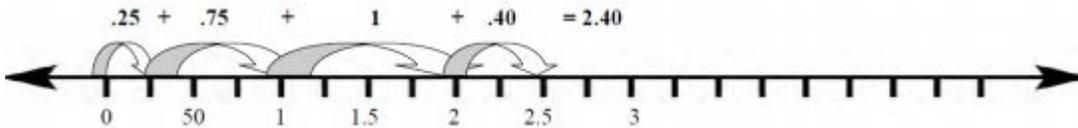
First, I broke the numbers apart. I broke 1.25 into $1.00 + 0.20 + 0.05$. I left 0.40 like it was. I broke 0.75 into $0.70 + 0.05$.

I combined my two 0.05's to get 0.10. I combined 0.40 and 0.20 to get 0.60. I added the 1 whole from 1.25. I ended up with 1 whole, 6 tenths, 7 more tenths, and another 1 tenths, so the total is 2.4.



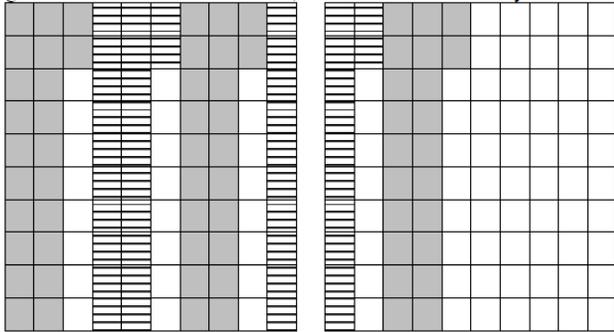
Student 2:

I saw that the 0.25 in the 1.25 cups of milk and the 0.75 cups of water would combine to equal 1 whole cup. That plus the 1 whole in the 1.25 cups of milk gives me 2 whole cups. Then I added the 2 wholes and the 0.40 cups of oil to get 2.40 cups.



Example of Multiplication 1

A gumball costs \$0.22. How much do 5 gumballs cost? Estimate the total, and then calculate. Was your estimate close?

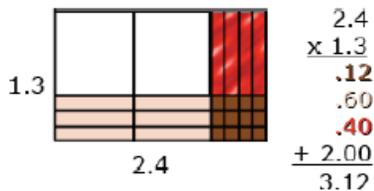


I estimate that the total cost will be a little more than a dollar. I know that 5 20's equal 100 and we have 5 22's. I have 10 whole columns shaded and 10 individual boxes shaded. The 10 columns equal 1 whole. The 10 individual boxes equal 10 hundredths or 1 tenth. My answer is \$1.10.

My estimate was a little more than a dollar, and my answer was \$1.10. I was really close.

Multiplication Example 2:

An area model can be useful for illustrating products.



Students should be able to describe the partial products displayed by the area model.

For example, “ $\frac{3}{10}$ times $\frac{4}{10}$ is $\frac{12}{100}$.

$\frac{3}{10}$ times 2 is $\frac{6}{10}$ or $\frac{60}{100}$.

1 group of $\frac{4}{10}$ is $\frac{4}{10}$ or $\frac{40}{100}$.

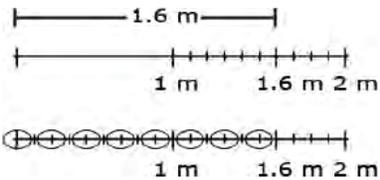
1 group of 2 is 2.”

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Division Example 1: Joe has 1.6 meters of rope. He has to cut pieces of rope that are 0.2 meters long. How many can he cut?

Finding the number of groups

Students could draw a segment to represent 1.6 meters. In doing so, s/he would count in tenths to identify the 6 tenths, and be able to identify the number of 2 tenths within the 6 tenths. The student can then extend the idea of counting by tenths to divide the one meter into tenths and determine that there are 5 more groups of 2 tenths.



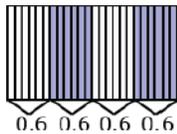
Students might count groups of 2 tenths without the use of models or diagrams. Knowing that 1 can be thought of as $\frac{10}{10}$, a student might think of 1.6 as 16 tenths. Counting 2 tenths, 4 tenths, 6 tenths, up to 16 tenths, a student can count 8 groups of 2 tenths.

Use their understanding of multiplication and think, “8 groups of 2 is 16, so 8 groups of $\frac{2}{10}$ is $\frac{16}{10}$ or $1\frac{6}{10}$.”

Division Example 2: $2.4 \div 4$

Finding the number in each group or share

Students should be encouraged to apply a fair sharing model separating decimal values into equal parts such as $2.4 \div 4 = 0.6$.

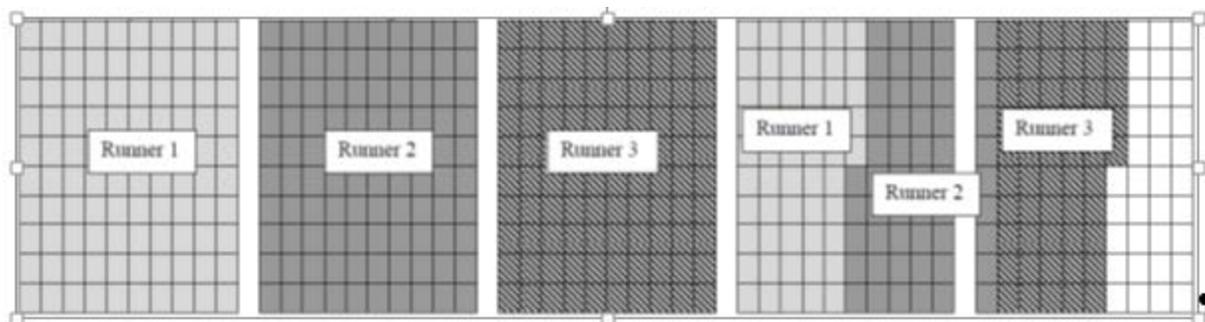


Division Example 3:

A relay race lasts 4.65 miles. The relay team has 3 runners. If each runner goes the same distance, how far does each team member run? Make an estimate, find your actual answer, and then compare them.

A possible solution is shown on the next page.

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My estimate is that each runner runs between 1 and 2 miles. If each runner went 2 miles, that would be a total of 6 miles which is too high. If each runner ran 1 mile, that would be 3 miles, which is too low. I used the 5 grids above to represent the 4.65 miles. I am going to use all of the first 4 grids and 65 of the squares in the 5th grid. I have to divide the 4 whole grids and the 65 squares into 3 equal groups. I labeled each of the first 3 grids for each runner, so I know that each team member ran at least 1 mile. I then have 1 whole grid and 65 squares to divide up. Each column represents one-tenth. If I give 5 columns to each runner, that means that each runner has run 1 whole mile and 5 tenths of a mile. Now, I have 15 squares left to divide up. Each runner gets 5 of those squares. So each runner ran 1 mile, 5 tenths and 5 hundredths of a mile. I can write that as 1.55 miles.

My answer is 1.55 and my estimate was between 1 and 2 miles. I was pretty close.

Module 2 - Arithmetic to Algebra

Students will extend arithmetic operations to algebraic modeling.

Expressions: An expression is a record of a computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function. Conventions about the use of parentheses and the order of operations assure that each expression is unambiguous. Creating an expression that describes a computation involving a general quantity requires the ability to express the computation in general terms, abstracting from specific instances.

Reading an expression with comprehension involves analysis of its underlying structure. This may suggest a different but equivalent way of writing the expression that exhibits some different aspect of its meaning. For example, $p + 0.05p$ can be interpreted as the addition of a 5% tax to a price p . Rewriting $p + 0.05p$ as $1.05p$ shows that adding a tax is the same as multiplying the price by a constant factor.

Algebraic manipulations are governed by the properties of operations and exponents, and the conventions of algebraic notation. At times, an expression is the result of applying operations to simpler expressions. For example, $p + 0.05p$ is the sum of the simpler expressions p and $0.05p$.

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Viewing an expression as the result of operation on simpler expressions can sometimes clarify its underlying structure.

A spreadsheet or a computer algebra system (CAS) can be used to experiment with algebraic expressions, perform complicated algebraic manipulations, and understand how algebraic manipulations behave.

MFAAA1. Students will generate and interpret equivalent numeric and algebraic expressions.

- a. Apply properties of operations emphasizing when the commutative property applies. ([MGSE7.EE.1](#))
- b. Use area models to represent the distributive property and develop understandings of addition and multiplication (all positive rational numbers should be included in the models). (MGSE3.MD.7)
- c. Model numerical expressions (arrays) leading to the modeling of algebraic expressions. ([MGSE7.EE.1,2](#) , [MGSE9-12.A.SSE.1,3](#))
- d. Add, subtract, and multiply algebraic expressions. ([MGSE6.EE.3,4](#), [MGSE7.EE.1](#), [MGSE9-12.A.SSE.3](#))
- e. Generate equivalent expressions using properties of operations and understand various representations within context. *For example, distinguish multiplicative comparison from additive comparison. Students should be able to explain the difference between “3 more” and “3 times”.* (MGSE4.OA.2; [MGSE6.EE.3](#), [MGSE7.EE.1,2](#) ,[MGSE9-12.A.SSE.3](#))
- f. Evaluate formulas at specific values for variables. *For example, use formulas such as $A = l \times w$ and find the area given the values for the length and width.* ([MGSE6.EE.2](#))

MFAAA2. Students will interpret and use the properties of exponents.

- a. Substitute numeric values into formulas containing exponents, interpreting units consistently. ([MGSE6.EE.2](#), [MGSE9-12.N.Q.1](#), [MGSE9-12.A.SSE.1](#), [MGSE9-12.N.RN.2](#))
- b. Use properties of integer exponents to find equivalent numerical expressions. *For example, $3^2 \times 3^{-5} = 3^{-3} = \frac{1}{3^3} = \frac{1}{27}$.* ([MGSE8.EE.1](#))
- c. Evaluate square roots of perfect squares and cube roots of perfect cubes ([MGSE8.EE.2](#))
- d. Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. ([MGSE8.EE.2](#))
- e. Use the Pythagorean Theorem to solve triangles based on real-world contexts (Limit to finding the hypotenuse given two legs). ([MGSE8.G.7](#))

Elementary Reference Standards

MGSE.3.MD.7 Relate area to the operations of multiplication and addition.

a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.

Students should tile rectangles, then multiply their side lengths to show it is the same.

To find the area, one could count the squares or multiply

$$3 \times 4 = 12.$$

1	2	3	4
5	6	6	8
9	10	11	12

b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.

Students should solve real world and mathematical problems

Example:

Drew wants to tile the bathroom floor using 1 foot tiles. How many square foot tiles will he need?

6 feet



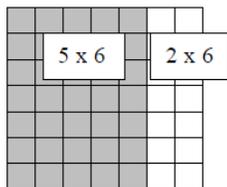
8 feet

The area of the rectangle is 48 square feet, and since each tile is 1 square foot, 48 tiles will be needed.

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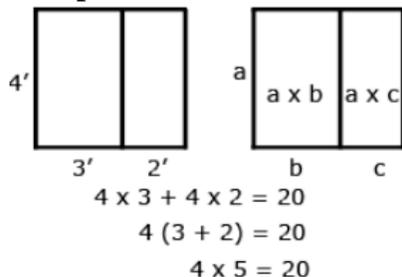
c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning.

This standard extends students' work with the distributive property. For example, in the picture below the area of a 7×6 figure can be determined by finding the area of a 5×6 and 2×6 and adding the two sums.



So, $7 \times 6 = (5 + 2) \times 6 = 5 \times 6 + 2 \times 6 = 30 + 12 = 42$

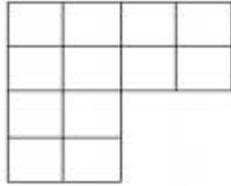
Example:



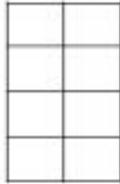
c. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.

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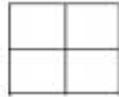
This standard uses the word rectilinear. A rectilinear figure is a polygon that has all right angles.



How could this figure be decomposed to help find the area?



This portion of the decomposed figure is a 4×2 .



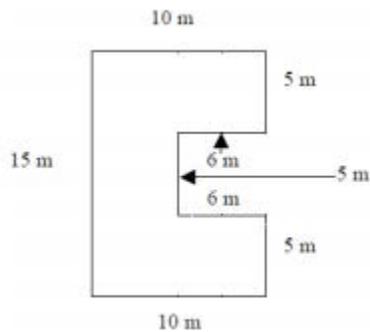
This portion of the decomposed figure is 2×2 .

$$4 \times 2 = 8 \text{ and } 2 \times 2 = 4$$
$$\text{So } 8 + 4 = 12$$

Therefore the total area of this figure is 12 square units

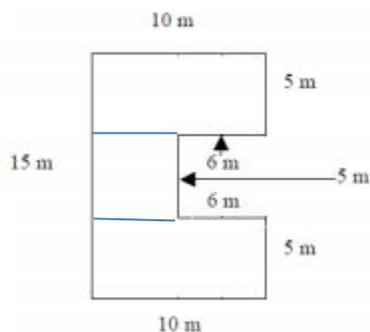
Example 1:

A storage shed is pictured below. What is the total area? How could the figure be decomposed to help find the area?



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The area can be found by using 3 rectangles:



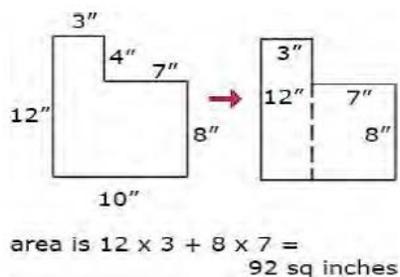
The top and bottom of the figure will be $10\text{ m} \times 5\text{ m}$ for $50\text{ m}^2 \times 2 = 100\text{ m}^2$

The center rectangle will be a square with dimensions $5\text{ m} \times (15 - 5 - 5)\text{ m} = 5\text{ m} \times 5\text{ m}$ or 25 m^2

So, the area of the storage shed is 125 square meters.

Example 2:

As seen above, students can decompose a rectilinear figure into different rectangles. They find the area of the figure by adding the areas of each of the rectangles together.



Common Misconceptions

Students may confuse perimeter and area when they measure the sides of a rectangle and then multiply. They think the attribute they find is length, which is perimeter. Pose problems situations that require students to explain whether they are to find the perimeter or area.

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MGSE.4.OA.2. Multiply or divide to solve word problems involving multiplicative comparison. Use drawings and equations with a symbol or letter for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.

This standard calls for students to translate comparative situations into equations with an unknown and solve. Students need many opportunities to solve contextual problems.

Examples:

Unknown Product: A blue scarf costs \$3. A red scarf costs 6 times as much. How much does the red scarf cost? ($3 \times 6 = p$)

Group Size Unknown: A book costs \$18. That is 3 times more than a DVD. How much does a DVD cost? ($18 \div p = 3$ or $3 \times p = 18$)

Number of Groups Unknown: A red scarf costs \$18. A blue scarf costs \$6. How many times as much does the red scarf cost compared to the blue scarf? ($18 \div 6 = p$ or $6 \times p = 18$)

When distinguishing multiplicative comparison from additive comparison, students should note the following:

- Additive comparisons focus on the difference between two quantities. For example, Deb has 3 apples and Karen has 5 apples. How many more apples does Karen have? A simple way to remember this is, “How many more?”
- Multiplicative comparisons focus on comparing two quantities by showing that one quantity is a specified number of times larger or smaller than the other. For example, Deb ran 3 miles. Karen ran 5 times as many miles as Deb. How many miles did Karen run? A simple way to remember this is “How many times as much?” or “How many times as many?”

Module 3 - Proportional Reasoning

Students will use ratios to solve real-world and mathematical problems.

Proportional reasoning is a process that requires instruction and practice. It does not develop over time on its own. Sixth grade is the first of several years in which students develop this multiplicative thinking. Examples with ratio and proportion must involve measurements, prices and geometric contexts, as well as rates of miles per hour or portions per person within contexts that are relevant to sixth graders. Experience with proportional and non-proportional relationships, comparing and predicting ratios, and relating unit rates to previously learned unit fractions will facilitate the development of proportional reasoning. Although algorithms provide

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efficient means for finding solutions, the cross-product algorithm commonly used for solving proportions will not aid in the development of proportional reasoning. Delaying the introduction of rules and algorithms will encourage thinking about multiplicative situations instead of indiscriminately applying rules.

MFAPR1. Students will explain equivalent ratios by using a variety of models. For example, tables of values, tape diagrams, bar models, double number line diagrams, and equations. (MGSE6.RP.3)

MFAPR2. Students will recognize and represent proportional relationships between quantities.

- Relate proportionality to fraction equivalence and division. *For example, $\frac{3}{6}$ is equal to $\frac{4}{8}$ because both yield a quotient of $\frac{1}{2}$ and, in both cases, the denominator is double the value of the numerator. (MGSE4.NF.1)*
- Understand real-world rate/ratio/percent problems by finding the whole given a part and find a part given the whole. (MGSE6.RP. 1,2,3, , MGSE7.RP.1,2)
- Use proportional relationships to solve multistep ratio and percent problems. (MGSE7.RP.2,3)

MFAPR3. Students will graph proportional relationships.

- Interpret unit rates as slopes of graphs. (MGSE8.EE.5)
- Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane. (MGSE8.EE.6)
- Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed. (MGSE8.EE.5)*

Elementary Reference Standards

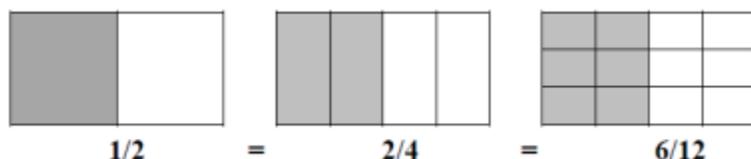
MGSE.4.NF.1 Explain why two or more fractions are equivalent. $\frac{a}{b} = \frac{n \times a}{n \times b}$. $\frac{1}{4} = \frac{3 \times 1}{3 \times 4}$ by using visual fraction models. Focus attention on how the number and size of the parts differ even though the fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

This standard refers to visual fraction models. This includes area models or number lines. This standard extends the work in third grade by using additional denominators (5, 10, 12, and 100).

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The standard addresses equivalent fractions by examining the idea that equivalent fractions can be created by multiplying both the numerator and denominator by the same number or by dividing a shaded region into various parts.

Example:



Technology Connection: <http://illuminations.nctm.org/activitydetail.aspx?id=80>

Module 4 - Equations and Inequalities

Students will solve, interpret, and create linear models using equations and inequalities.

Equations and inequalities: An equation is a statement of equality between two expressions, often viewed as a question asking for which values of the variables result in the expressions on either side being equal. These values are the solutions to the equation. An identity, in contrast, is true for all values of the variables; identities are often developed by rewriting an expression in an equivalent form.

The solutions of an equation in one variable form a set of numbers; the solutions of an equation in two variables form a set of ordered pairs of numbers, which can be plotted in the coordinate plane. Two or more equations and/or inequalities form a system. A solution for such a system must satisfy every equation and inequality in the system.

An equation can often be solved by successively deducing from it one or more simpler equations. For example, one can add the same constant to both sides without changing the solutions, but squaring both sides might lead to extraneous solutions. Strategic competence in solving includes looking ahead for productive manipulations and anticipating the nature and number of solutions.

The same solution techniques used to solve equations can be used to rearrange formulas. For example, the formula for the area of a trapezoid, $A = \frac{b_1 + b_2}{2} h$, can be solved for h using the same deductive process. Inequalities can be solved by reasoning about the properties of inequality. Many, but not all, of the properties of equality continue to hold for inequalities and can be useful in solving them.

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MFAEI1. Students will create and solve equations and inequalities in one variable.

- Use variables to represent an unknown number in a specified set. ([MGSE.6.EE.2,5,6](#))
- Explain each step in solving simple equations and inequalities using the equality properties of numbers. ([MGSE9-12.A.REI.1](#))
- Construct viable arguments to justify the solutions and methods of solving equations and inequalities. ([MGSE9-12.A.REI.1](#))
- Represent and find solutions graphically.
- Use variables to solve real-world and mathematical problems. ([MGSE6.EE.7](#),[MGSE7.EE.4](#))

MFAEI2. Students will use units as a way to understand problems and guide the solutions of multi-step problems.

- Choose and interpret units in formulas. ([MGSE9-12.N.Q.1](#))
- Choose and interpret graphs and data displays, including the scale and comparisons of data. (MGSE3.MD.3, [MGSE9-12.N.Q.1](#))
- Graph points in all four quadrants of the coordinate plane. ([MGSE6.NS.8](#))

MFAEI3. Students will create algebraic models in two variables.

- Create an algebraic model from a context using two-variable equations. ([MGSE6.EE.6](#), [MGSE8.EE.8](#), [MGSE9-12.A.CED.2](#))
- Find approximate solutions using technology to graph, construct tables of values, and find successive approximations. ([MGSE9-12.A.REI.10,11](#))
- Represent solutions to systems of equations graphically or by using a table of values. ([MGSE6.EE.5](#); [MGSE7.EE.3](#); [MGSE8.EE.8](#); [MGSE9-12.A.CED.2](#))
- Analyze the reasonableness of the solutions of systems of equations within a given context. ([MGSE6.EE.5,6](#), [MGSE7.EE.4](#))

MFAEI4. Students will solve literal equations.

- Solve for any variable in a multi-variable equation. ([MGSE6.EE.9](#),[MGSE9-12.A.REI.3](#))
- Rearrange formulas to highlight a particular variable using the same reasoning as in solving equations. *For example, solve for the base in $A = \frac{1}{2}bh$.* ([MGSE9-12.A.CED.4](#))

Elementary Reference Standards

MGSE.3.MD.3 Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step “how many more” and “how many less” problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.*

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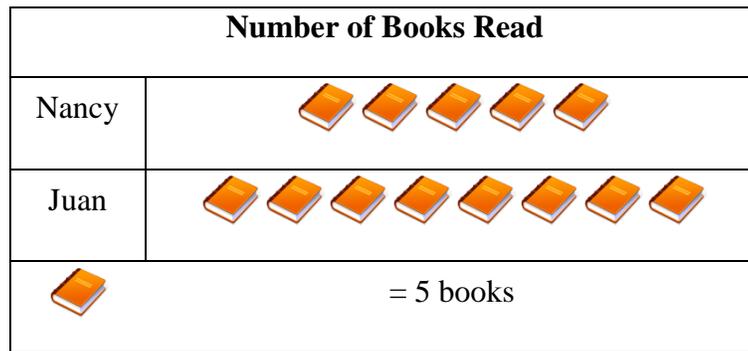
Students should have opportunities to read and solve problems using scaled graphs before being asked to draw one. The following graphs all use five as the scale interval, but students should experience different intervals to further develop their understanding of scale graphs and number facts. While exploring data concepts, students should **Pose** a question, **Collect** data, **Analyze** data, and **Interpret** data. Students should be graphing data that is relevant to their lives

Example:

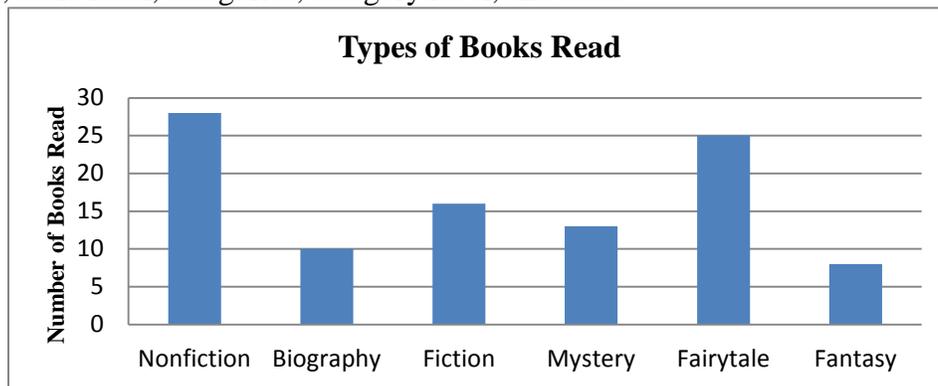
Pose a question: Student should come up with a question. What is the typical genre read in our class?

Collect and organize data: student survey

Pictographs: Scaled pictographs include symbols that represent multiple units. Below is an example of a pictograph with symbols that represent multiple units. Graphs should include a title, categories, category label, key, and data. How many more books did Juan read than Nancy?



Single Bar Graphs: Students use both horizontal and vertical bar graphs. Bar graphs include a title, scale, scale label, categories, category label, and data.



Analyze and Interpret data:

- How many more nonfiction books were read than fantasy books?
- Did more people read biography and mystery books or fiction and fantasy books?
- About how many books in all genres were read?
- Using the data from the graphs, what type of book was read more often than a mystery but less often than a fairytale?
- What interval was used for this scale?
- What can we say about the types of books read? What is a typical type of book read?
- If you were to purchase a book for the class library, which would be the best genre? Why?

Module 5 - Quantitative Reasoning with Functions

Students will create function statements and analyze relationships among pairs of variables using graphs, table, and equations.

Connections to Functions: Expressions can define functions, and equivalent expressions can define the same function. Asking when two functions have the same value for the same input leads to an equation; graphing the two functions allows for finding approximate solutions of the equation. Converting a verbal description to an equation, inequality, or system of these is an essential skill in modeling.

Functions describe situations where one quantity determines another. For example, the return on \$10,000 invested at an annualized percentage rate of 4.25% is a function of the length of time the money is invested. Because we continually make theories about dependencies between quantities in nature and society, functions are important tools in the construction of mathematical models. In school, functions usually have numerical inputs and outputs and are often defined by an algebraic expression. For example, the time in hours it takes for a car to drive 100 miles is a function of the car's speed in miles per hour, v ; the rule $T(v) = \frac{100}{v}$ expresses this relationship algebraically and defines a function whose name is T .

The set of inputs to a function is called its domain. We often infer the domain to be all inputs for which the expression defining a function has a value, or for which the function makes sense in a given context.

A function can be described in various ways, such as by a graph (e.g., the trace of a seismograph); by a verbal rule, as in, "I'll give you a state, you give me the capital city"; by an algebraic expression like $f(x) = a + bx$; or by a recursive rule. The graph of a function is often a useful way of visualizing the relationship of the function models, and manipulating a mathematical expression for a function can throw light on the function's properties.

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Functions presented as expressions can model many important phenomena. Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate. Linear functions with a constant term of zero describe proportional relationships.

Connections to Expressions, Equations, Modeling, and Coordinates:

Determining an output value for a particular input involves evaluating an expression; finding inputs that yield a given output involves solving an equation. Questions about when two functions have the same value for the same input lead to equations whose solutions can be visualized from the intersection of their graphs. Because functions describe relationships between quantities, they are frequently used in modeling.

MFAQR1. Students will understand characteristics of functions.

- a. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. ([MGSE9-12.F.IF.1](#))
- b. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.* ([MGSE9-12.F.IF.5](#))
- c. Graph functions using sets of ordered pairs consisting of an input and the corresponding output. ([MGSE8.F.1.2](#))

MFAQR2. Students will compare and graph functions.

- a. Calculate rates of change of functions, comparing when rates increase, decrease, or stay constant. *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.* ([MGSE6.RP.2](#); [MGSE7.RP.1,2,3](#), [MGSE8.F.2,5](#), [MGSE9-12.F.IF.6](#))
- b. Graph by hand simple functions expressed symbolically (use all four quadrants). ([MGSE9-12.F.IF.7](#))
- c. Interpret the equation $y = mx + b$ as defining a linear function whose graph is a straight line. ([MGSE8.F.3](#))
- d. Use technology to graph non-linear functions. ([MGSE8.F.3](#), [MGSE9-12.F.IF.7](#))
- e. Analyze graphs of functions for key features (intercepts, intervals of increase/decrease, maximums/minimums, symmetries, and end behavior) based on context. ([MGSE9-12.F.IF.4,7](#))
- f. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear*

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function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the great rate of change.

([MGSE8.F.2](#))

MFAQR3. Students will construct and interpret functions.

- a. Write a function that describes a relationship between two quantities.
([MGSE8.F.4](#), [MGSE9-12.F.BF.1](#))
- b. Use variables to represent two quantities in a real-world problem that change in relationship to one another (conceptual understanding of a variable). ([MGSE6.EE.9](#))
- c. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of context. ([MGSE9-12.F.IF.2](#))

Breakdown of a Scaffolded Instructional Lesson

How do I go about tackling a lesson or a module? Foundations of Algebra modules have lessons with several different parts and suggestions.

1. **Read the module in its entirety.** Discuss the module with your grade level colleagues. Which parts do you feel comfortable with? Which make you wonder? Brainstorm ways to implement the lessons. Collaboratively complete the culminating lesson with your grade level colleagues. As students work through the lessons, the teacher will be able to facilitate their learning with this end in mind.
2. **Read the first lesson** students will be engaged in. Discuss it with grade level colleagues. Which parts do you feel comfortable with? Which make you wonder? Brainstorm ways to implement the lesson.
3. Each lesson is comprised of an **opener/activator, work session, and closer**. If not already established, use the first few weeks of school to establish routines and rituals, and to assess student mathematical understanding.

In the **Foundations of Algebra modules**, lessons have differentiation ideas and interventions; some of these are from the Global Assessment of Strategy Stages (GloSS) <http://nzmaths.co.nz/gloss-forms> . These are listed in the Interventions Tables in the first four modules. Many of the lessons have extension and enrichment activities. Each teacher and team will make the modules their own and add to them to meet the needs of the learners. A revisit of the *Essential Questions*, which are listed at the beginning of every lesson, can help ensure that the content and standards are being encompassed throughout the lesson.

Routines and Rituals

The importance of continuing the established classroom routines cannot be overstated. Daily routines must include such obvious activities as **estimating, analyzing data, describing patterns, and answering daily questions**. They should also include less obvious routines, such as how to select materials, how to use materials in a productive manner, how to put materials away, how to access classroom technology such as computers and calculators. An additional suggested routine is to allow plenty of time for students to explore new materials before attempting any directed activity with these new materials. The regular use of routines is important to the development of students' number sense, flexibility, fluency, collaborative skills, and communication. These routines contribute to a rich, hands-on standards based classroom and will support students' performances on the lessons in this unit and throughout the school year.

GSE Effective Instructional Practices Guide

The Modules for Foundations of Algebra incorporate many instructional practices. Included in these practices are Three Act Tasks, Formative Instructional Practices, and Number Talks. Detailed information about all of these can be found in the GSE Effective Instructional Practices guide <http://bit.ly/1FrYid5>

Three-Act Tasks

A Three-Act Task is a whole group mathematics task consisting of 3 distinct parts: an engaging and perplexing Act One, an information and solution seeking Act Two, and a solution discussion and solution revealing Act Three.

Formative Assessment Lessons

The **Formative Assessment Lesson** is designed to be part of an instructional unit typically implemented approximately two-thirds of the way through the instructional module. Formative Assessment Lessons are intended to support teachers in formative assessment. They both reveal and develop students' understandings of key mathematical ideas and applications. These lessons enable teachers and students to monitor in more detail their progress towards the targets of the standards. They assess student understanding of important concepts and problem solving performance, and the lessons help teachers and their students work effectively together to move each student's mathematical reasoning forward.

Videos of Georgia Teachers implementing Formative Assessment Lessons can be accessed at <http://bit.ly/1ALzGiw>

Number Talks

A Number Talk is a 10 to 15 minute whole group mental mathematics activity where students find the answer to a mathematics problem in their heads, then students share aloud the strategies they used to find that answer. This strategy helps to develop quality student discourse in a whole class setting as students are encouraged to explain their thinking, justify their reasoning, and make sense of each other's strategies.

A number talk resource for middle grade topics can be found at http://mathematicssolutions.com/wp-content/uploads/14_nctm_kcohen_numbertalks.pdf

For high school, topics can be explored at this site authored by a Georgia Educator: <http://gfletchy.com/2014/07/22/on-you-marks-get-set-number-talks/>

Strategies for Teaching and Learning

- Students should be actively engaged by developing their own understanding.
- Mathematics should be represented in as many ways as possible by using graphs, tables, pictures, symbols and words.
- Interdisciplinary and cross curricular strategies should be used to reinforce and extend the learning activities.
- Appropriate manipulatives and technology should be used to enhance student learning.
- Students should be given opportunities to revise their work based on teacher feedback, peer feedback, and metacognition which includes self-assessment and reflection.
- Students should write about the mathematical ideas and concepts they are learning.
- Consideration of all students should be made during the planning and instruction of a module.

Teachers need to consider the following:

- What level of support do my struggling students need in order to be successful with this unit? **Am I utilizing the Intervention Table to enable student success?**
- In what way can I deepen the understanding of those students who are competent in this unit? **Am I utilizing the Intervention Table to enhance student success?**
- What real life connections can I make that will help my students utilize the skills practiced in this unit?

Specific strategies for effective teaching and learning are as follows:

Teaching Mathematics in Context and Through Problems

Traditionally, mathematics instruction has been centered on many problems in a single mathematics lesson and focused on rote procedures and algorithms which do not promote conceptual understanding. Teaching through word problems and in context is difficult; however, there are excellent reasons for making the effort.

- Problem solving focuses a student's attention on ideas and sense making.
- Problem solving develops the belief in students that they are capable of doing the mathematics and that mathematics makes sense.
- Problem solving provides ongoing assessment data.
- Problem solving is an excellent method for attending to a breadth of abilities.
- Problem solving engages students so that there are fewer discipline problems.
- Problem solving develops "mathematical power" (Van de Walle, John. Teaching Student-Centered Mathematics: Developmentally Appropriate Instruction for Grades 3 – 5. Pearson: 2013, 15 -16)

Please see the problem solving rubric below to help incorporate contextual problems.

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Problem Solving Rubric

SMP	1-Emergent	2-Progressing	3- Meets/Proficient	4-Exceeds
Make sense of problems and persevere in solving them.	The student was unable to explain the problem and showed minimal perseverance when identifying the purpose of the problem.	The student explained the problem and showed some perseverance in identifying the purpose of the problem, and selected and applied an appropriate problem solving strategy that lead to a partially complete and/or partially accurate solution.	The student explained the problem and showed perseverance when identifying the purpose of the problem, and selected an applied and appropriate problem solving strategy that lead to a generally complete and accurate solution.	The student explained the problem and showed perseverance by identifying the purpose of the problem and selected and applied an appropriate problem solving strategy that lead to a thorough and accurate solution. In addition, student will check answer using another method.
Attends to precision	The student was unclear in their thinking and was unable to communicate mathematically.	The student was precise by clearly describing their actions and strategies, while showing understanding and using appropriate vocabulary in their process of finding solutions.	The student was precise by clearly describing their actions and strategies, while showing understanding and using grade-level appropriate vocabulary in their process of finding solutions.	
Reasoning and explaining	The student was unable to express or justify their opinion quantitatively or abstractly using numbers, pictures, charts or words.	The student expressed or justified their opinion either quantitatively OR abstractly using numbers, pictures, charts OR words.	The student expressed and justified their opinion both quantitatively and abstractly using numbers, pictures, charts and/or words. Student is able to make connections between models and equations.	The student expressed and justified their opinion both quantitatively and abstractly using a variety of numbers, pictures, charts and words. The student connects quantities to written symbols and create a logical representation with precision.
Models and use of tools	The student was unable to select an appropriate tool, draw a representation to reason or justify their thinking.	The student selected an appropriate tools or drew a correct representation of the tools used to reason and justify their response.	The student selected an efficient tool and/or drew a correct representation of the efficient tool used to reason and justify their response.	The student selected multiple efficient tools and correctly represented the tools to reason and justify their response. In addition this students was able to explain why their tool/ model was efficient
Seeing structure and generalizing	The student was unable to identify patterns, structures or connect to other areas of mathematics and/or real-life.	The student identified a pattern or structure in the number system and noticed connections to other areas of mathematics or real-life.	The student identified patterns or structures in the number system and noticed connections to other areas of mathematics and real-life.	The student identified various patterns and structures in the number system and noticed connections to multiple areas of mathematics and real-life.

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Journaling

The use of a notebook or folder is a great way to organize the mathematical thinking occurring in the classroom and gives students a place to record answers to problems. The journal entries can be from Module lessons but should also include all mathematical thinking; student's journal entries demonstrate their thinking processes. Journal entries should be simple to begin with and become more detailed as the student's problem-solving skills improve. Students should always be allowed to discuss their representations with classmates if they desire feedback. Each entry could first be shared with a "buddy" to encourage discussion and explanation; then one or two students could share their entries with the entire class. Do not forget to praise students for their thinking skills and their journal entries! These journals are perfect for assessment and for parent conferencing because the student's thinking is made visible!

When beginning math journals, the teacher might consider modeling the process initially, showing students how to organize the pages for easy access. Discuss the usefulness of the book, and the way in which the journal will help students retrieve their mathematical thinking whenever they need it.

Most journaling suggestions in Foundations of Algebra occur at the close of a lesson. During the closing, students could share their journals with the class using a document camera or overhead. This is an excellent opportunity to discuss different ways to organize thinking and clarity of explanations.

"Students should be writing and talking about math topics every day. Putting thoughts into words helps to clarify and solidify thinking. By sharing their mathematical understandings in written and oral form with their classmates, teachers, and parents, students develop confidence in themselves as mathematical learners; this practice also enables teachers to better monitor student progress." (State of New Jersey: Department of Education. *Model Curriculum*. Web 10 May 2015.)

"Language, whether used to express ideas or to receive them, is a very powerful tool and should be used to foster the learning of mathematics. Communicating about mathematical ideas is a way for students to articulate, clarify, organize, and consolidate their thinking. Students, like adults, exchange thoughts and ideas in many ways—orally; with gestures; and with pictures, objects, and symbols. By listening carefully to others, students can become aware of alternative perspectives and strategies. By writing and talking with others, they learn to use more-precise mathematical language and, gradually, conventional symbols to express their mathematical ideas. Communication makes mathematical thinking observable and therefore facilitates further development of that thought. It encourages students to reflect on their own knowledge and their own ways of solving problems. Throughout the early years, students should have daily opportunities to talk and write about mathematics." (National Council of Teachers of Mathematics. *Mathematics in Early Childhood Learning*. Web 10 May 2015)

Mathematics Manipulatives

"Use of Manipulatives Used correctly, manipulatives can be a positive factor in children's learning. It is important that you have a good perspective on how manipulatives can help or fail to help children construct ideas." (Van de Walle and Lovin, Teaching Student-Centered Mathematics: 3-5 pg. 6) When a new model with new use is introduced into the classroom, it is generally a good idea to explain how the model is used and perhaps conduct a simple activity that illustrates this use. Once you are comfortable that the models have been explained, you should not force their use on students. Rather, students should feel free to select and use models that make sense to them. In most instances, not using a model at all should also be an option. The choice a student makes can provide you with valuable information about the level of sophistication of the student's reasoning. Whereas the free choice of models should generally be the norm in the classroom, you can often ask students to model to show their thinking. This will help you find out about a child's understanding of the idea and also his or her understanding of the models that have been used in the classroom.

The following are simple rules of thumb for using models:

- Introduce new models by showing how they can represent the ideas for which they are intended.
- Allow students (in most instances) to select freely from available models to use in solving problems.
- Encourage the use of a model when you believe it would be helpful to a student having difficulty. (Van de Walle and Lovin, Teaching Student-Centered Mathematics 3-5, pg. 9)
- Modeling also includes the use of mathematical symbols to represent/model the concrete mathematical idea/thought process/situation.

This is a very important, yet often neglected step along the way. Modeling can be concrete, representational, and abstract. Each type of model is important to student understanding. Modeling also means to "mathematize" a situation or problem, to take a situation which might at first glance not seem mathematical, and view it through the lens of mathematics. For example, students notice that the cafeteria is always out of their favorite flavor of ice cream on ice cream days. They decide to survey their schoolmates to determine which flavors are most popular, and share their data with the cafeteria manager so that ice cream orders reflect their findings. The problem: Running out of ice cream flavors. The solution: Use math to change the flavor amounts ordered."

Number Lines

Number lines should be used in **all grade levels** and in multiple settings. Familiarity with a number line is essential to strengthening students' ability to think fluidly and flexibly about numbers. To be successful in mathematics students need to have a sense that numbers are meaningful, i.e. what numbers mean and how they are related to one another. Identifying where a particular number is on a number line and figuring out the relationship other numbers have to a given number will help increase students overall number sense understandings.

Third grade students use number lines to locate fractions and add and subtract time intervals; fourth graders locate decimals and use number lines for measurement, and fifth graders use perpendicular number lines in coordinate grids (Council of Chief State School Offices, *Tool to Enhance State Mathematics College and Career Readiness Standards Implementation Plans*, 2010). Middle and high school students use number lines to compare, order, and graph real numbers.

Open number lines are particularly useful for building number sense. They can also form the basis for discussions that require the precise use of vocabulary and quantities, and are therefore a good way to engage students in the Standards of Mathematical Practice.

Games

“A game or other repeatable activity may not look like a problem, but a game can nonetheless be problem-based. The determining factor is this: Does the activity cause students to be reflective about new or developing (math) relationships? If the activity merely has students repeating procedure without wrestling with an emerging idea, then it is not a problem-based experience. Students playing a game can keep records and then tell about how they played the game- what thinking or strategies they used.” (Van de Walle, John. Teaching Student-Centered Mathematics: Developmentally Appropriate Instruction for Grades 3 – 5. Pearson: 2013, 28)

Technology Links

Georgiastandards.org provides a gateway to a wealth of instruction links and information. Open the GSE Mathematics to access specific GSE resources for this course.

The assessments for Foundations of Algebra can be found through the Georgia Online Formative Assessment Resource (GOFAR). <http://bit.ly/1ALzGiw>

[Georgia Virtual School](#) content available on the Shared Resources Website is available for anyone to view. Courses are divided into modules and are aligned with the Georgia Standards of Excellence.

[Course/Grade Level WIKI](#) spaces are available to post questions about a unit, a standard, the course, or any other GSE mathematics related concern. Shared resources and information are also available at the site.

Websites Referenced in the Modules

Teacher resources and professional development across the curriculum <http://www.learner.org/> .

Instructional and assessment tasks, lesson plans and other resources. <https://www.illustrativemathematics.org/>

Professional resource for teachers <http://www.insidemathematics.org/>

Vocabulary Sites: https://www.youtube.com/watch?v=Rbts9h_ruu8 (Term, Information, Picture (TIP charts), <https://wvde.state.wv.us/strategybank/FrayerModel.html> (Frayer Models) <http://www.amathsdictionaryforkids.com/>
<http://intermath.coe.uga.edu/dictionary/homepg.asp>.

Games, Problems, and videos <http://www.mathplayground.com/>

Activities and lesson plans <http://illuminations.nctm.org/>

Lessons, games, and more <http://www.homeschoolmath.net/>

Conceptualization of fractions <http://www.visualfractions.com/>

Resources for students <https://learnzillion.com/>

Classroom Manipulatives www.visualpatterns.org.
http://www.glencoe.com/sites/common_assets/mathematics/ebook_assets/vmf/VMF-Interface.html
http://www.ct4me.net/math_manipulatives_2.htm

Resources Consulted

Boaler, Jo. <http://joboaler.com/>

Georgia Department of Education. *Mathematics Georgia Standards of Excellence*. <http://bit.ly/1AJddmx>

Georgia Department of Education. *Testing/Assessment*.

Inside Mathematics. <http://www.insidemathematics.org/> <http://www.gadoe.org/Curriculum-Instruction-and-Assessment/Assessment/Pages/default.aspx>

Math Solutions. *Journaling*. <http://mathsolutions.com/teachers/>

KATM Resources: *Elementary Flip Books* <http://bit.ly/1wJD0ZR>

National Council of Teacher of Mathematics. <http://www.nctm.org/>

Van de Walle, John. Teaching Student-Centered Mathematics: Developmentally Appropriate Instruction for Grades 3 – 5. Pearson: 2013.