Name _____ Date _____ Class _____

LESSON 3-5

Practice C

Factoring Polynomials

Use the Factor Theorem to verify that each linear binomial is a factor of the given polynomial. Then use synthetic division to write the polynomial as a product.

1. $(x + 5)$; $P(x) = 2x^2 + 6x - 20$

2. $(x - 1)$; $P(x) = x^4 - 6x^3 + 4x^2 + 1$

3. $(x + 2)$; $P(x) = 3x^3 + 12x^2 + 17x + 10$

4. $(x - 8)$; $P(x) = x^4 - 8x^3 - 4x^2 + 33x - 8$

Factor each expression.

5. $16x^3 - 12x^2 + 20x - 15$

6. $3x^6 + 54x^4 + 243x^2$

7. $x^6 - 10x^5 + 25x^4$

8. $6x^3 + 12x^2 + 4x + 8$

9. $250x^4 + 54x$

10. $-3x^5 + 24x^2$

Solve.

11. The voltage generated by an electrical circuit changes over time according to the polynomial $V(t) = t^3 - 4t^2 - 25t + 100$, where V is in volts and t is in seconds. Factor the polynomial to find the times when the voltage is equal to zero.

8. $-x^3 - 10x^2 - 24x - 72 - \frac{217}{x-3}$

9. $P(5) = 438$ 10. $P(-2) = -79$

11. $2t^2 + 100$

Review for Mastery

1. $4x - 1 + \frac{8}{x+2}$ 2. $2x + 1 + \frac{5}{x+4}$

3. $3x + 10$ 4. $2x + 1 - \frac{8}{3x+2}$

5. $4x - 1 + \frac{12}{x+2}$

6. $a = 5$

$2x + 4 + \frac{8}{x-5}$

Challenge

1. $2x - 2 + \frac{1}{x+3}$

2. $x^2 + 7x + 16 + \frac{53}{x-3}$

3. $5x^2 - 10x + 26 - \frac{44}{x+2}$

4. $\frac{1}{2}x^3 + \frac{1}{4}x^2 + \frac{1}{8}x + \frac{1}{16}$

5. $x - 2 + \frac{5}{2x-1}$ 6. $x + \frac{2}{3} - \frac{16\frac{1}{3}}{3x+5}$

7. $x^4 + 2x^3 + 4x^2 + 8x + 16 - \frac{1}{4x-8}$

8. $x^4 + 9x^2 + 81$

Problem Solving

1. $\frac{\sqrt{3}}{4}x^2$

2. B 3. C

4. A 5. D

Reading Strategies

1. Multiply divisor and quotient and add remainder, and see if it equals the dividend. $x(3x + 6) + 2 = 3x^2 + 6x + 2$

2. a. $x + 3 + \frac{1}{x+1}$

   b. $(x + 3)$ R1

   c. $x + 1\overline{)x^2 + 4x + 4}$ $\begin{array}{r} x+3 \\ \\ \text{R1} \end{array}$

   d. $(x + 1)(x + 3) + 1$

3. No; the degree of the divisor has to be less than the degree of the dividend.

4. The product of the divisor and the quotient equals the dividend.

3-5 FACTORING POLYNOMIALS

Practice A

1. False 2. True

3. False 4. True

5. Yes 6. No

7. $2(x + 4)(x + 1)$

8. $(x + 2)(x + 1)(x - 1)$ 9. $(x^2 + 7)(x + 1)$

10. $(x + 2)(x - 2)(x - 2)$

11. $(g + 2)(g^2 - 2g + 4)$

12. $2m(4 - m)(16 + 4m + m^2)$

13. No; possible answer: the polynomial is the difference of two cubes; she used the formula for the sum of two cubes.

Practice B

1. Yes 2. No

3. Yes 4. No

5. $x(2x - 1)(x + 1)$ 6. $(4x + 1)(x^2 - 2)$

7. $(5x^3 + 1)(x^2 - 1)$

8. $2x(x + 3)(x^2 - 3x + 9)$

9. $(4x - 1)(16x^2 + 4x + 1)$

10. $3x(x + 2)(x^2 - 2x + 4)$

11. 2016; $-(x - 10)(x^2 - 6x + 14)$

Practice C

1. $(x + 5)(2x - 4)$

2. $(x - 1)(x^3 - 5x^2 - x - 1)$

3. $(x + 2)(3x^2 + 6x + 5)$

4. $(x - 8)(x^3 - 4x + 1)$ 5. $(4x - 3)(4x^2 + 5)$

6. $3x^2(x^2 + 9)(x^2 + 9)$ 7. $x^4(x - 5)^2$

8. $2(3x^2 + 2)(x + 2)$
9. $2x(5x + 3)(25x^2 - 15x + 9)$
10. $-3x^2(x - 2)(x^2 + 2x + 4)$
11. $(t - 4)(t - 5)(t + 5)$; the voltage is equal to zero at 4 s and 5 s.

Review for Mastery

1. $(x - 3)(x^2 - 4)$
   $(x - 3)(x + 2)(x - 2)$
2. $x^2(x + 6) - 1(x + 6)$
   $(x + 6)(x^2 - 1)$
   $(x + 6)(x + 1)(x - 1)$
3. $(x^3 + x^2) + (-9x - 9)$
   $x^2(x + 1) - 9(x + 1)$
   $(x + 1)(x^2 - 9)$
   $(x + 1)(x + 3)(x - 3)$
4. $(x^3 + 2x^2) + (-16x - 32)$
   $x^2(x + 2) - 16(x + 2)$
   $(x + 2)(x^2 - 16)$
   $(x + 2)(x + 4)(x - 4)$
5. $(3x + 2)(9x^2 - 6x + 4)$
6. $(y - 6)(y^2 + 6y + 36)$
7. $y^3 + 3^3$
   $(y + 3)(y^2 - 3y + 9)$
8. $x^3 - 1^3$
   $(x - 1)(x^2 + x + 1)$

Challenge

1. $\{[(6)x + 8]x - 5\}x + 1$
2. $(\{[(-2)x + 5]x - 1\}x + 3)x - 4$
3. $N(3) = 128$ 4. $N(3) = 128$
5. $N(x) = 5x^3 - 3x^2 + 7x - 1$
6. $N(3) = 5 \cdot 3^3 - 3 \cdot 3^2 + 7 \cdot 3 - 1 = 128$
7. $P(x) = \{[(2)x - 6]x - 2\}x - 30$,
   $P(4) = \{[(2)4 - 6]4 - 2\}4 - 30$
   $= [(2)4 - 2]4 - 30 = (6)4 - 30 = -6$; The numbers in the innermost nests are the coefficients of the quotient, and the last number is the remainder. So the quotient is $2x^2 + 2x + 6$ and the remainder is -6.
8. The quotient is $2x^2 + 2x + 6$ and the remainder is -6.

Problem Solving

1. a. $-2, 4, 6$
   b. $(x + 2)(x - 4)(x - 6)$
2. a. $-4, -1, 0$
   b. $(2x + 2)(x + 4)(x)$
3. a. 5, 8, 9
   b. $(x - 5)(x - 8)(x - 9)$
4.

Basket	Dimensions (in terms of x)	Actual Dimensions	Volume
A	$(x - 5)$, $(x - 8)$, $(x - 9)$	7 by 4 by 3	84 cubic units
B	$(x + 2)$, $(x - 4)$, $(x - 6)$	14 by 8 by 6	672 cubic units
C	$(2x + 2)$, $(x + 4)$, (x)	26 by 16 by 12	4992 cubic units

5. No; the dimensions of each basket are doubled from one size to the next except for 14 to 26.
6. No; $\frac{84}{672} \neq \frac{672}{4992}$

Reading Strategies

1. Multiply $(x + 3)$ and $(x^2 + 2)$.
2. No; there are no two factors that have $x^2 + 2$ as their product.
3. 5
4. a. $(x^3 - 8x^2) + (-x + 8)$
   b. x^2
   c. -1
   d. $x^2(x - 8) - 1(x - 8) = (x - 8)(x^2 - 1)$
   e. $x^2 - 1$; $(x + 1)(x - 1)$
   f. $(x - 8)(x + 1)(x - 1)$

4-1 FINDING REAL ROOTS OF POLYNOMIAL EQUATIONS

Practice A

1. $x^2 + 2x + 1$; $-1, 0$
2. $2x^2 - 18$; $-3, -2, 3$
3. $-5, 0$ 4. $-1, 0, 7$
5. $x = -3$ with multiplicity 1; $x = 0$ with multiplicity 1; $x = 1$ with multiplicity 1