

Academic Pre-Calculus

Chapter 4

Trigonometry

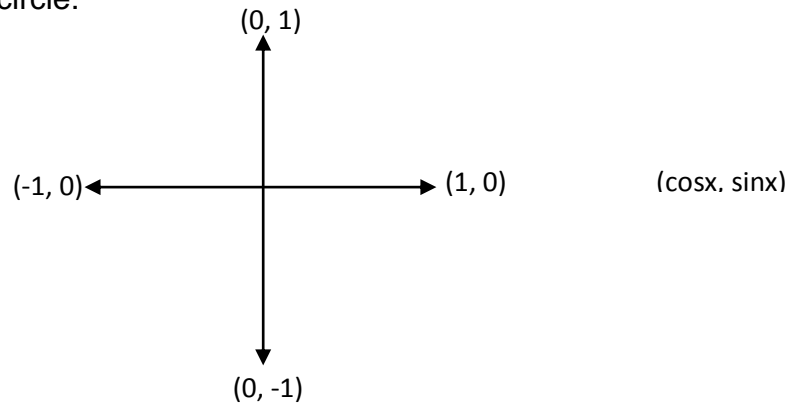
4.5 Graphs of Sine and Cosine Functions

4.6 Graphs of Other Trigonometric Functions

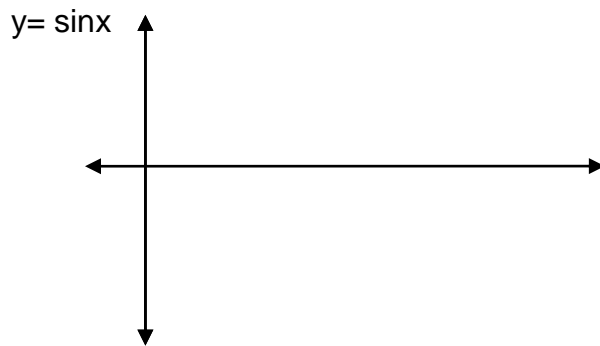
4.7 Inverse Trigonometric Functions

4.5 Day 1: Graphing Sine and Cosine Functions

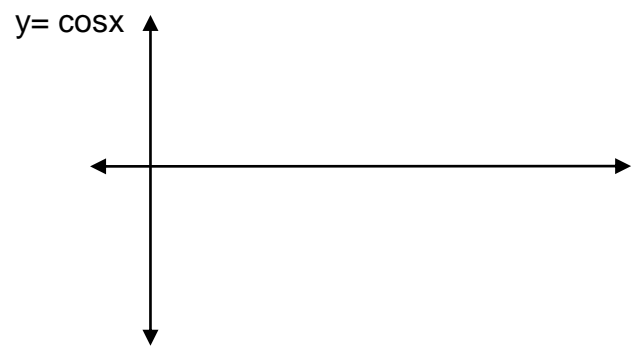
Remember from the unit circle:



So, the graph of $y = \sin\theta$ looks like:
like:



And, the graph of $y = \cos\theta$ looks



Given: $y = a\sin bx$ or $y = a\cos bx$

a = amplitude, which is the distance from the x-axis to the top of the graph (**always** positive!)

$-a$: reflects x-axis

b = number of cycles in a period from 0 to 2π

$-b$: reflects y-axis

$$\text{period} = \frac{2\pi}{b}$$

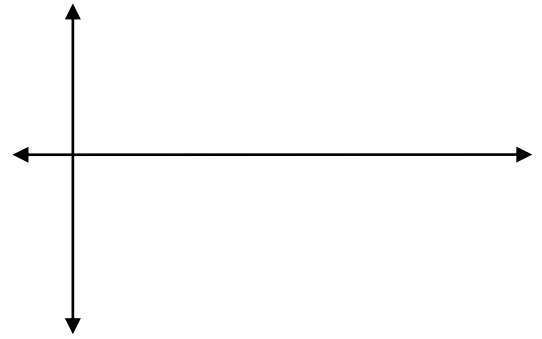
$$\text{increments} = \frac{\text{period}}{4}$$

Sketch the graph of each function for one period.

Determine the amplitude and period for the following trig functions.

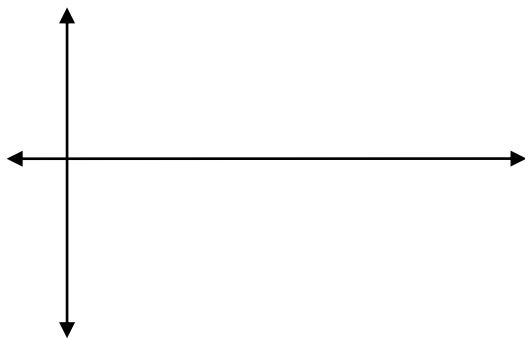
1) $y = 3\sin 4x$

2) $y = 5 \cos 2x$



3) $y = -4\sin\left(\frac{1}{4}x\right)$

4) $y = -\frac{1}{2}\cos x$



Name: _____

4.5 Day 1: Sine and Cosine

Transformations

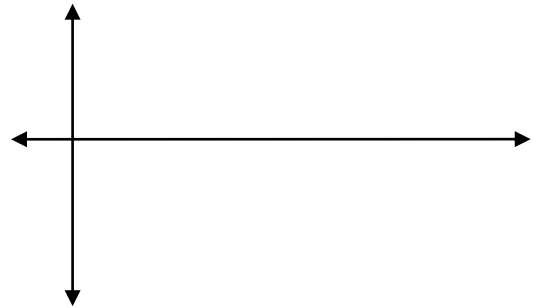
Homework

Sketch the graph of each function for one period.

Determine the amplitude and period for the following trig graphs.

1. $y = 2 \sin 3x$

2. $y = -2 \cos 2x$



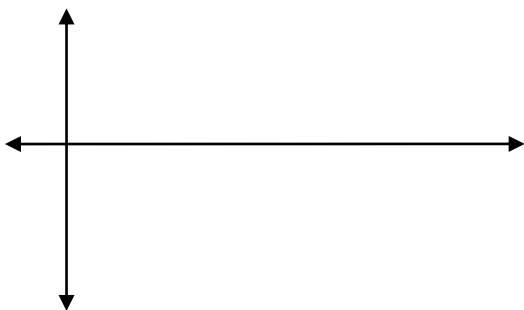
3. $y = 5 \cos 4x$

4. $y = -\sin(1/2x)$



5. $y = \cos 2x$

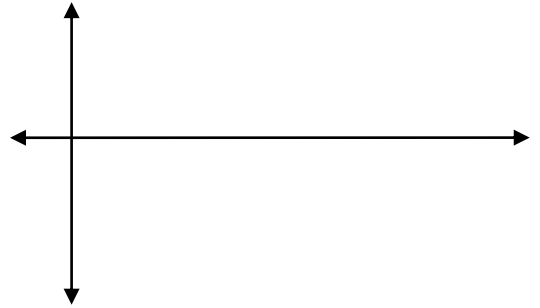
6. $y = -3 \sin x$



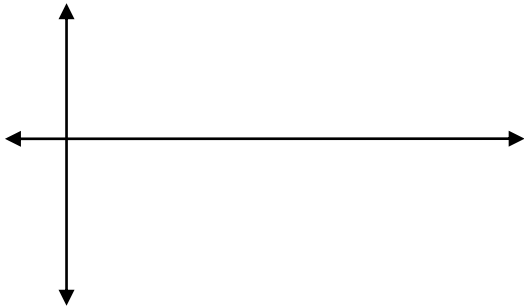
7. $y = \frac{1}{4} \sin 2x$



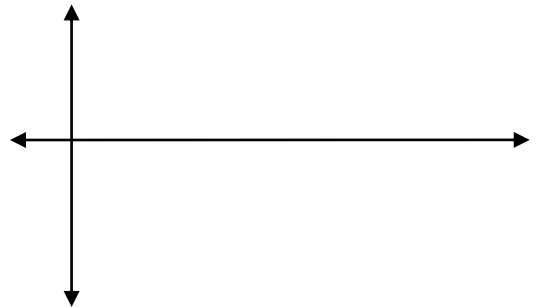
8. $y = -\cos x$



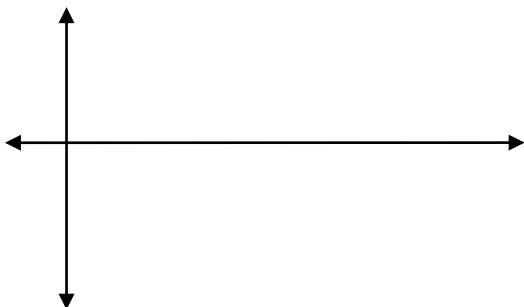
9. $y = 4 \cos (1/2) x$



10. $y = -2\sin 2x$



11. $y = -3\cos 2x$



12. $y = \frac{1}{2} \sin 2x$



Warm – Up

Describe the transformation:

1) $y = (x - 3)^2 + 5$

2) $y = (x + 4)^2 - 6$

Factor out the 3:

3) $y = (3x - 9)$

4) $y = (3x - 1)$

5) $y = \left(3x + \frac{1}{2}\right)$

4.5 Day 2: Graphing Sine and Cosine Functions

Given: $y = a \sin b(x \pm h) \pm k$ or $y = a \cos b(x \pm h) \pm k$

a = amplitude, which is the distance from the x-axis to the top of the graph (**always** positive!)

- a : reflects x axis

b = number of cycles in a period from 0 to 2π

- b : reflects y axis

h : phase shift (+ goes left, - goes right)

k : vertical shift (+ goes up, - goes down)

$$\text{period} = \frac{2\pi}{b}$$

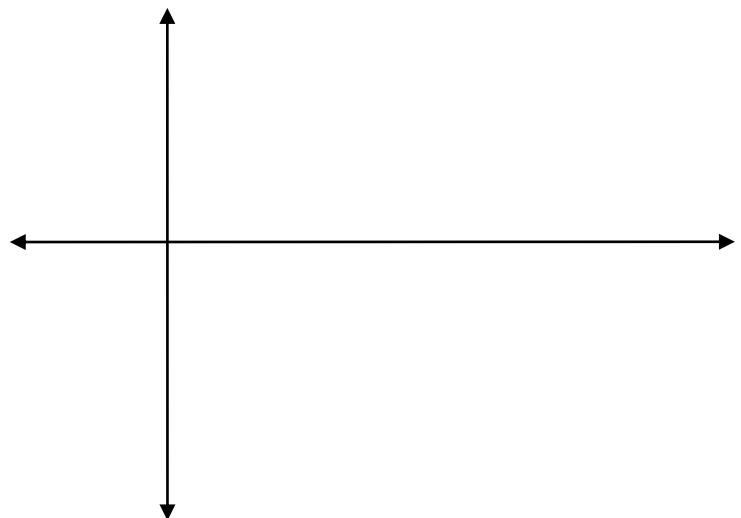
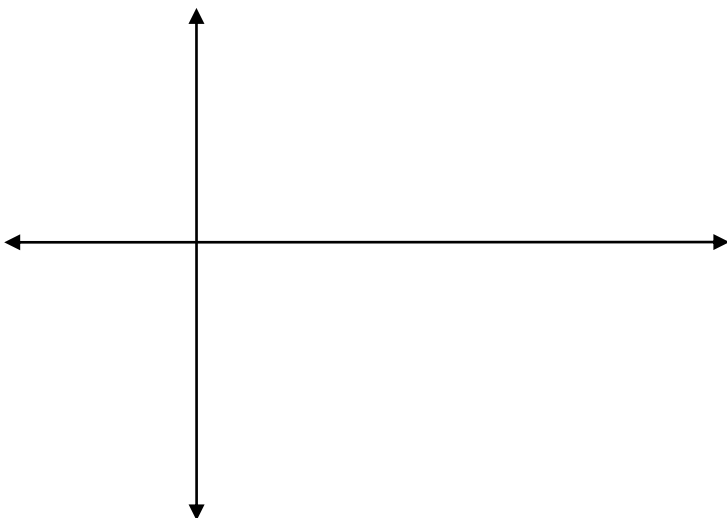
$$\text{increments} = \frac{\text{period}}{4}$$

Sketch the graph of each function for one period.

Determine the amplitude, period, phase shift and vertical shift for the following trig functions.

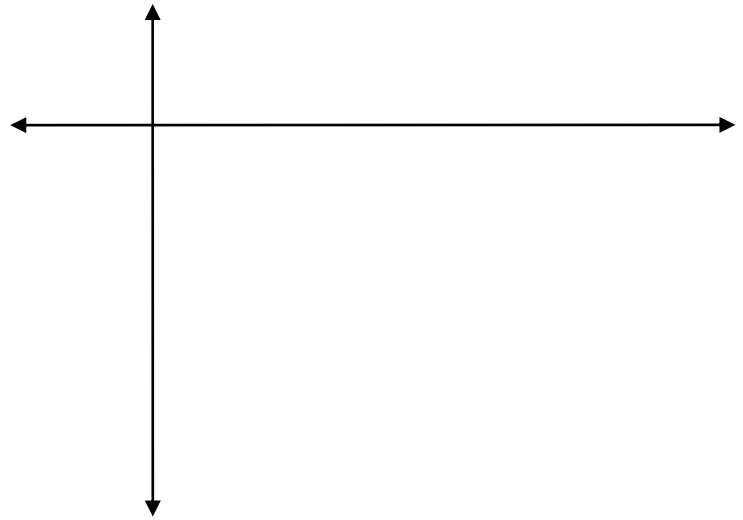
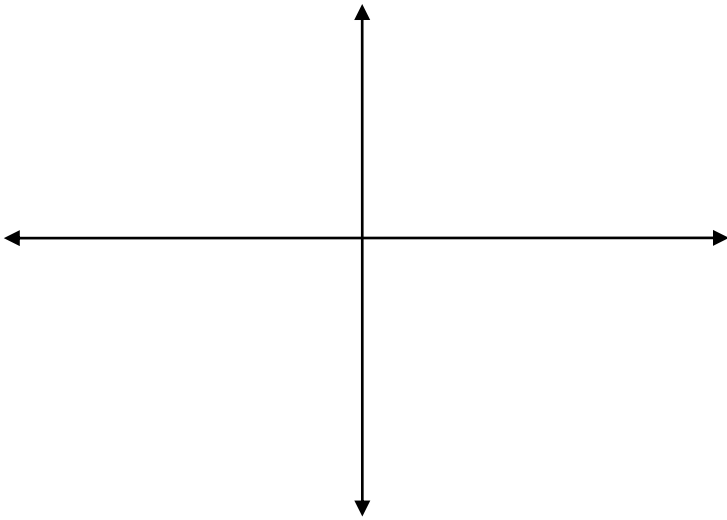
1) $y = 2 \cos 2 \left(x + \frac{\pi}{8} \right) + 3$

2) $y = 3 \sin 2 \left(x - \frac{\pi}{2} \right) - 2$



3) $y = -4 \cos(2x + \pi) + 1$

4) $y = -5 \sin\left(2x - \frac{\pi}{2}\right) - 6$



Name: _____

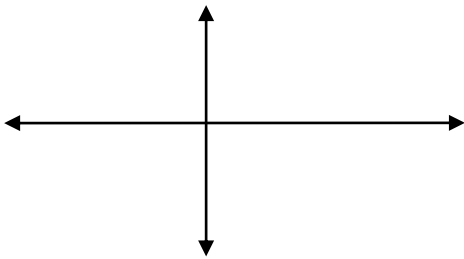
4.5 Day 2: Sine and Cosine Transformations
Homework

Sketch the graph of each function for one period.

Determine the amplitude, period, phase shift, and vertical shift for each.

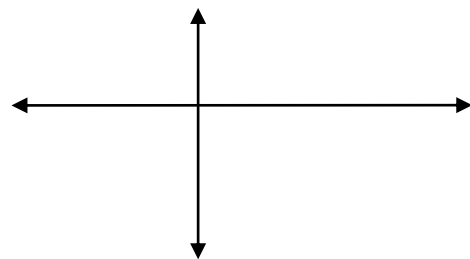
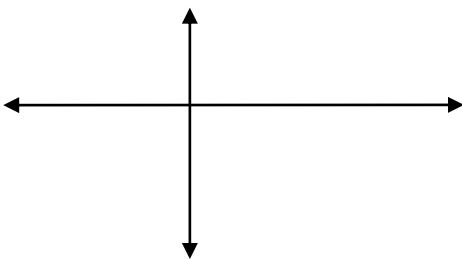
1. $y = 2 \sin (2x + 2\pi)$

2. $y = \cos 2x - 5$



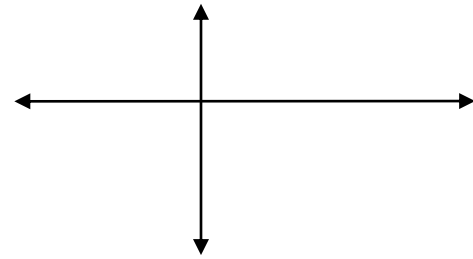
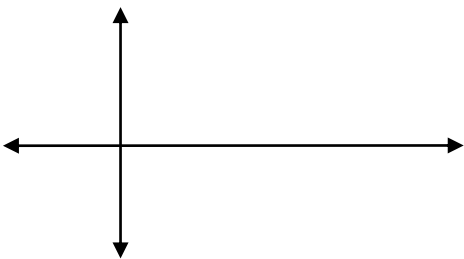
3. $y = \sin (x - \pi) - 1$

4. $y = 3 \sin x - 1$



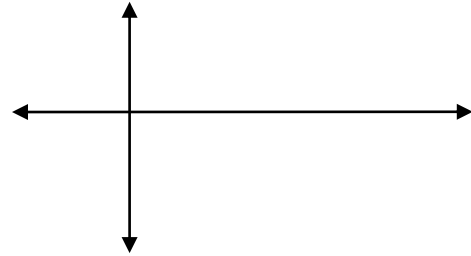
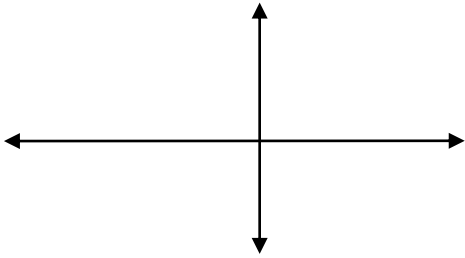
5. $y = 4 \sin (1/2)x + 2$

6. $y = 3 \sin (2x - \pi) - 1$



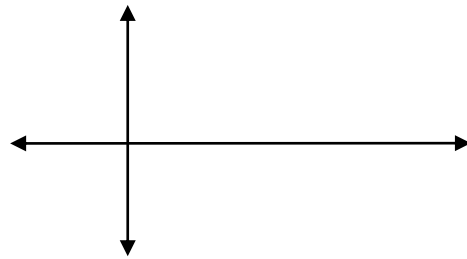
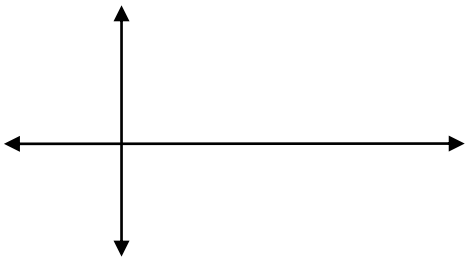
7. $y = \frac{1}{4} \sin(2x+2\pi)$

8. $y = -\cos x - 1$



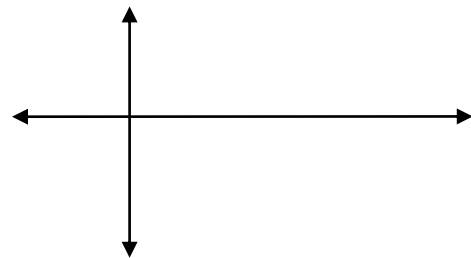
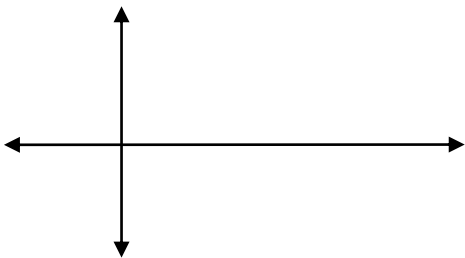
9. $y = 4 \sin\left(x - \frac{\pi}{3}\right) + 2$

10. $y = \cos 2(x - \pi)$



11. $y = 3 \sin\left(2x - \frac{\pi}{2}\right) + 1$

12. $y = \cos 2x - 1$

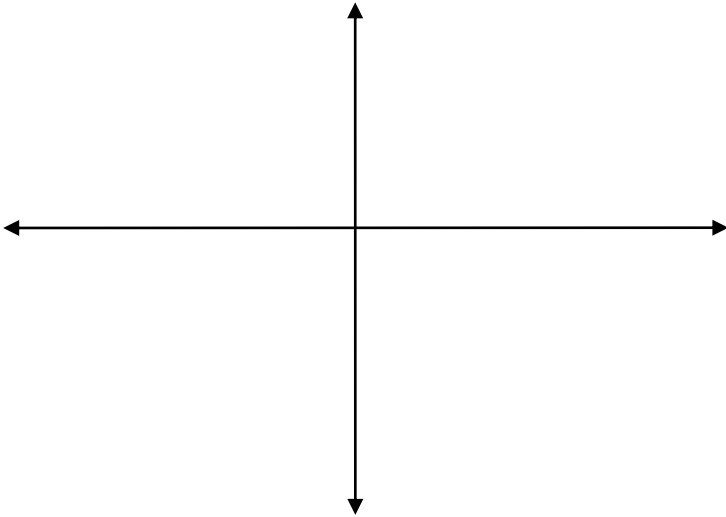


Warm Up

Sketch the graph of the function for one period.

Determine the amplitude, period, phase shift, and vertical shift for each.

$$y = 4\sin\left(3x + \frac{\pi}{4}\right) - 1$$

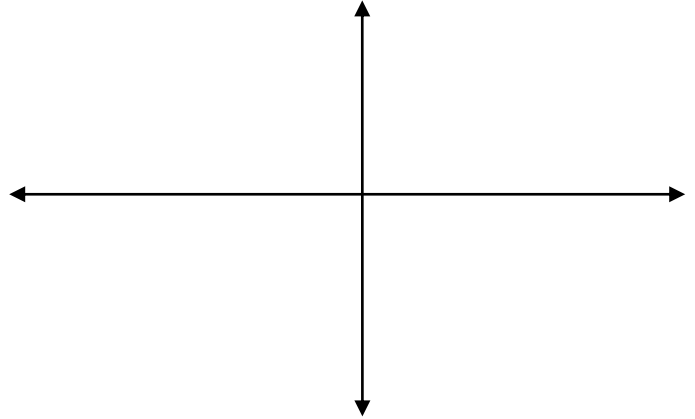


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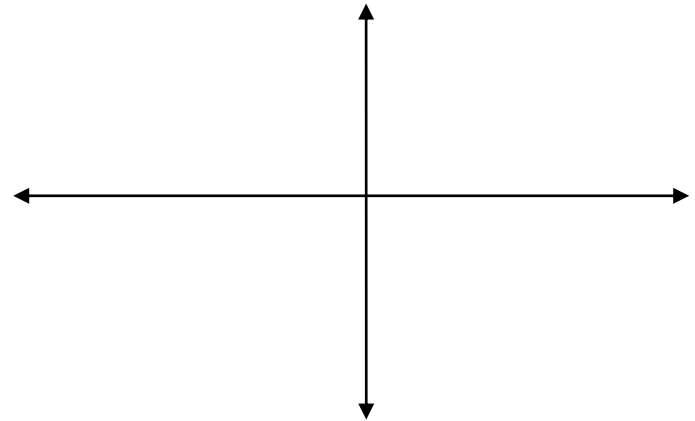
Worksheet: Review for sine/cosine transformations

Date: _____

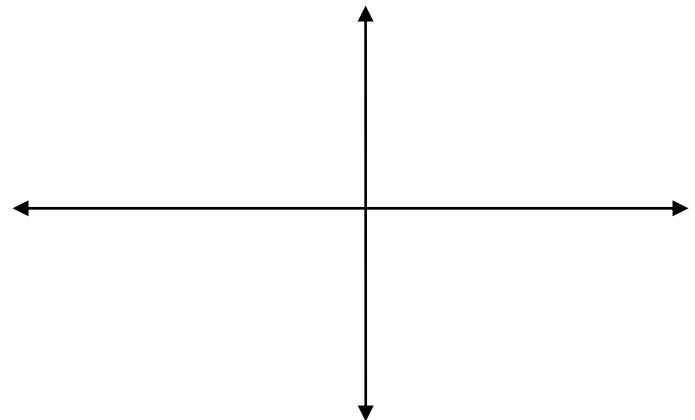
1. $y = 4\sin 2x$



2. $y = -\cos 2x$

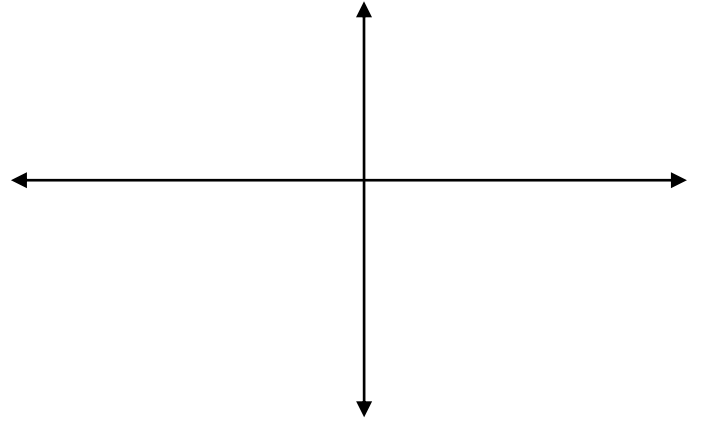


3. $y = -4\sin(1/2x)$



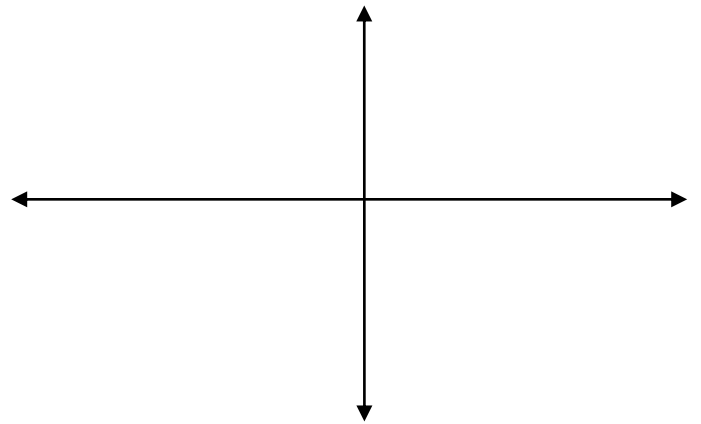
4.

$$y = \cos(2x - \pi) + 3$$



5.

$$y = \sin 2\left(x - \frac{3\pi}{4}\right)$$

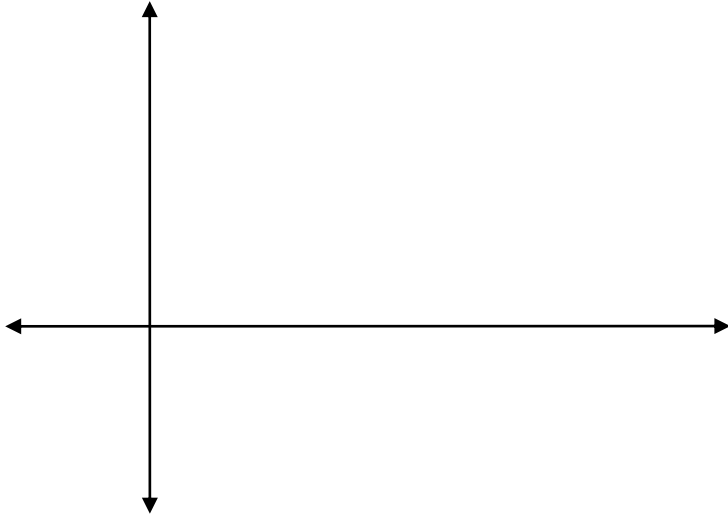


Warm Up

Sketch the graph of the function for one period.

Determine the amplitude, period, phase shift, and vertical shift for each.

$$y = -4\cos\left(\frac{1}{2}x - \pi\right) + 3$$



4.6 Day 1: Graphs of Secant and Cosecant

Remember from graphs of sin and cos:

$$\text{Given: } y = a \sin b(x \pm h) \pm k \quad \text{or} \quad y = a \cos b(x \pm h) \pm k$$

a = amplitude, which is the distance from the x-axis to the top of the graph (**always** positive!)

Secant and Cosecant graphs do not have amplitude. Instead, it is a vertical stretch.

$$\text{Given: } y = a \sec b(x \pm h) \pm k \quad \text{or} \quad y = a \csc b(x \pm h) \pm k$$

a = vertical stretch (**always** positive!)
to 2π

$-a$: reflects x axis

b = number of cycles in a period from 0

$-b$: reflects y axis

h : phase shift (+ goes left, - goes right)

k : vertical shift (+ goes up, - goes down)

$$\text{period} = \frac{2\pi}{b}$$

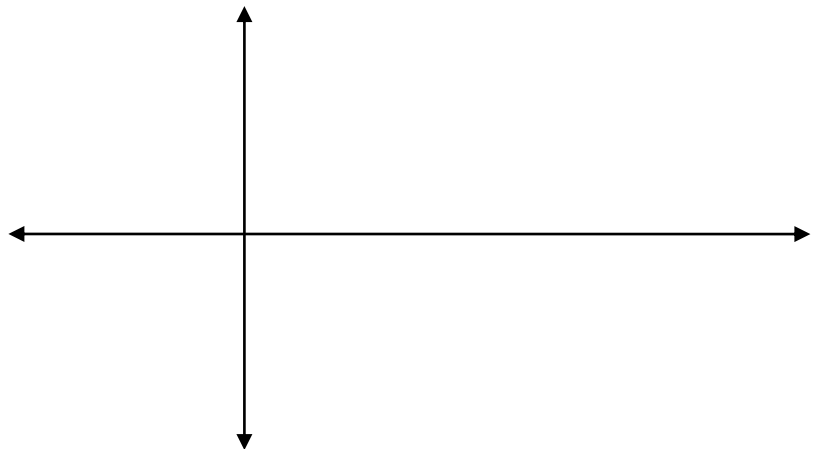
$$\text{increments} = \frac{\text{period}}{4}$$

Steps to graph secant or cosecant:

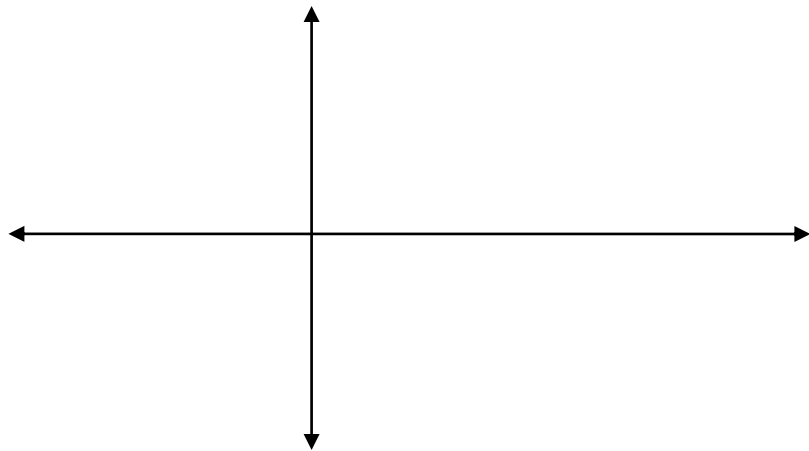
- 1) If csc, graph as if it were sin or if sec, graph as cos, but graph as a dotted line.
- 2) Graph vertical asymptotes: where the graph of sin or cos crosses the x-axis.
- 3) Go to each max and min vertex and graph the reciprocal (flip the graph).

Graph & determine the vertical stretch, period, phase shift, vertical shift and asymptotes.

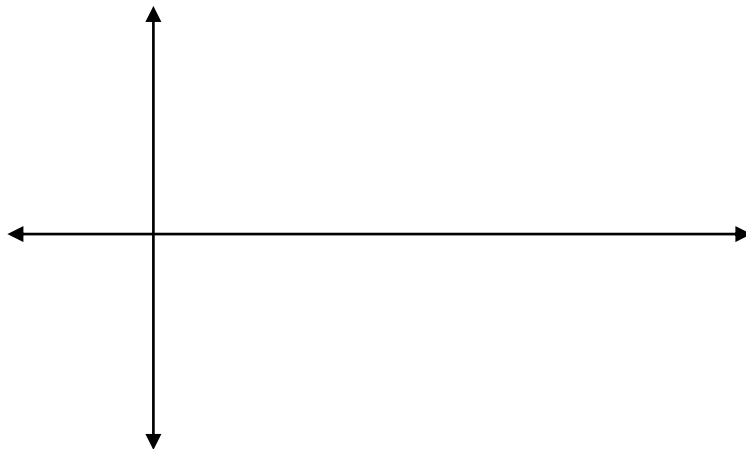
1) $y = 4 \csc 3(x - \pi)$



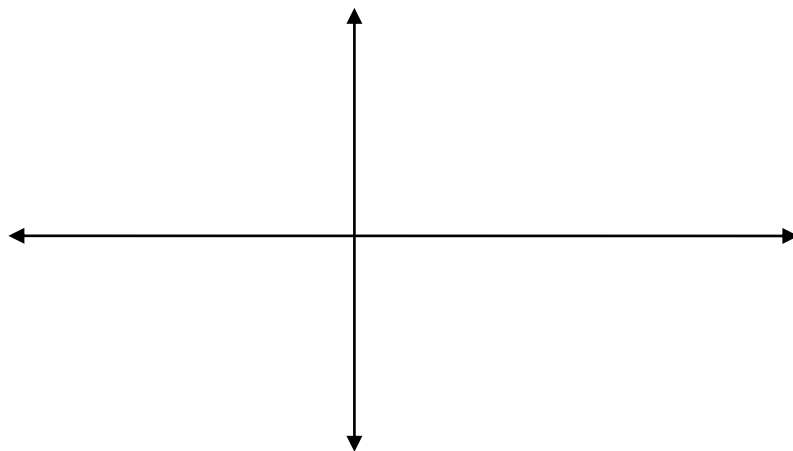
$$2) y = -2\sec\left(4x + \frac{3\pi}{4}\right) + 2$$



$$3) y = \sec\left(3x - \frac{2\pi}{3}\right) - 1$$



$$4) y = -3\csc\left(x + \frac{\pi}{4}\right) + 3$$



Name: _____

Period: _____ Date: _____

4.6 Day 1 Homework

GRAPHS OF SECANT AND COSECANT FUNCTIONS

I. Determine the period, phase shift, and vertical shift, if any, of each function.

1. $y = 4 \sec 3(x - \pi)$ Vertical Stretch _____ Period _____ Phase Shift _____ Vertical Shift _____

2. $y = 2 \csc 2(x + \pi)$ Vertical Stretch _____ Period _____ Phase Shift _____ Vertical Shift _____

3. $y = \sec\left(x - \frac{\pi}{4}\right)$ Vertical Stretch _____ Period _____ Phase Shift _____ Vertical Shift _____

4. $y = 3 \csc\left(x + \frac{2\pi}{3}\right)$ Vertical Stretch _____ Period _____ Phase Shift _____ Vertical Shift _____

5. $y = -2 \sec 2(x + \pi) + 3$ Vertical Stretch _____ Period _____ Phase Shift _____ Vertical Shift _____

6. $y = 6 \csc\left(3x + \frac{2\pi}{3}\right) - 2$ Vertical Stretch _____ Period _____ Phase Shift _____ Vertical Shift _____

7. $y = 2 \sec\left(4x + \frac{3\pi}{4}\right) + 3$ Vertical Stretch _____ Period _____ Phase Shift _____ Vertical Shift _____

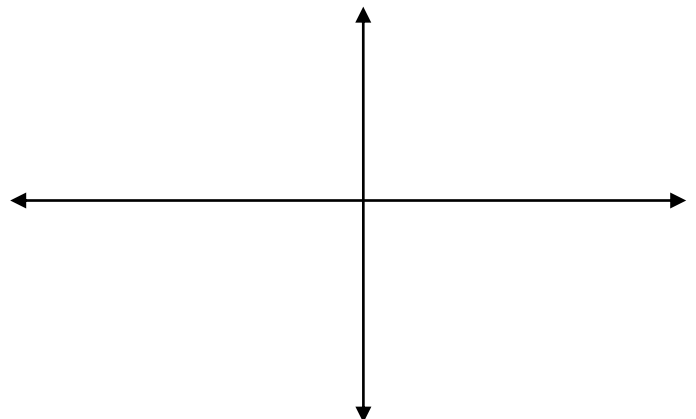
8. $y = 5 \csc 3(x - \pi) - 5$ Vertical Stretch _____ Period _____ Phase Shift _____ Vertical Shift _____

9. $y = \csc\left(x + \frac{5\pi}{6}\right) + 4$ Vertical Stretch _____ Period _____ Phase Shift _____ Vertical Shift _____

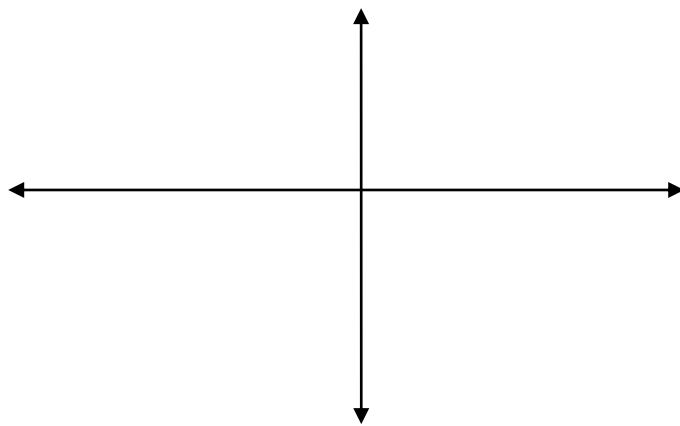
10. $y = -2 \sec\left(2x - \frac{3\pi}{2}\right) + 1$ Vertical Stretch _____ Period _____ Phase Shift _____ Vertical Shift _____

II. Graph each function over a one period interval. Make sure to label axes.

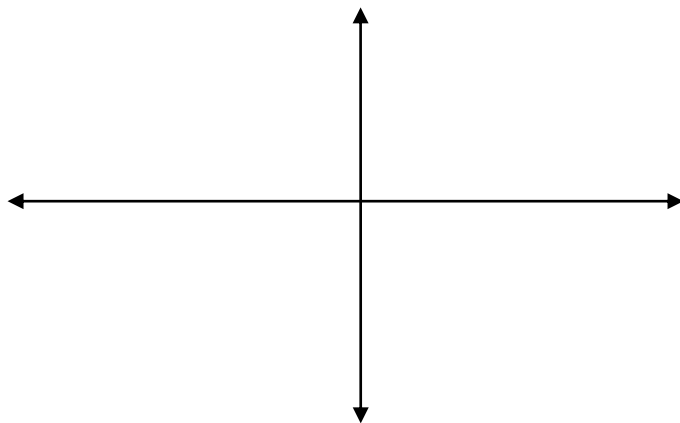
1. $y = 3 \csc 2x$



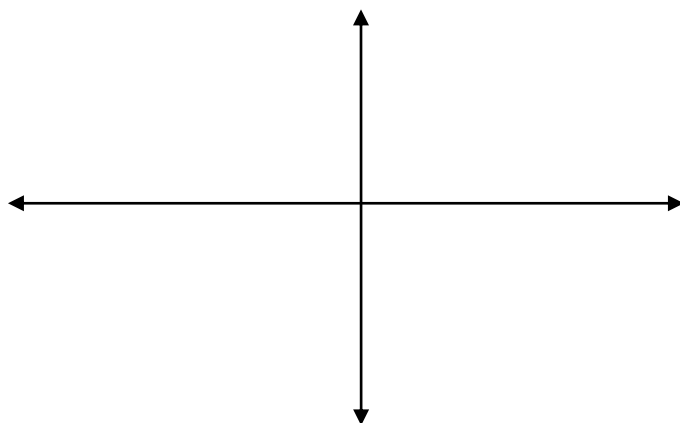
2. $y = 5 \sec 2x$



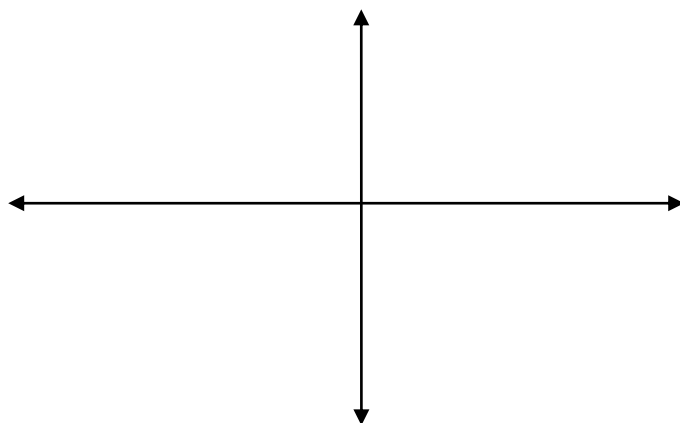
3. $y = 4 \csc 2x$



4. $y = 2 \sec(x - \pi)$



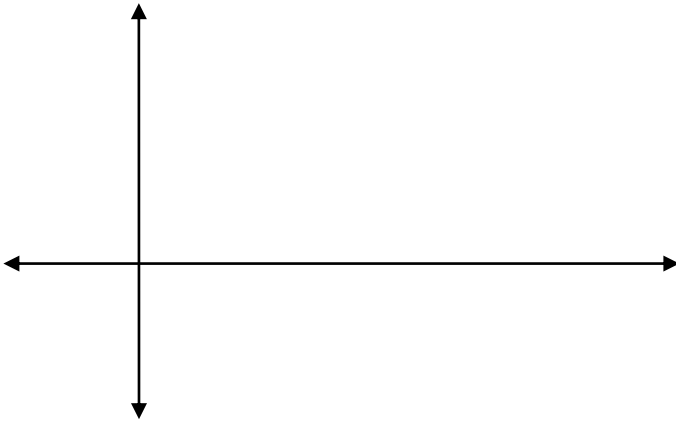
5. $y = 3 \csc 2\left(x - \frac{3\pi}{4}\right)$



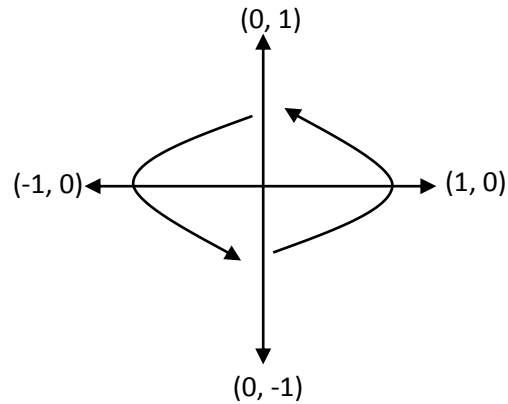
Warm Up

Graph each function over a one period interval. Make sure to label axes.

$$y = -2 \sec\left(2x - \frac{3\pi}{2}\right) + 1$$



4.6 Day 2: Graphing Tangent and Cotangent



Tan is undefined at

$$-\frac{\pi}{2}, \frac{\pi}{2} \text{ and } \frac{3\pi}{2}$$

Given: $y = a \tan b(x \pm h) \pm k$ or $y = a \cot b(x \pm h) \pm k$

a = vertical stretch ($a \geq 1$) or vertical shrink ($a < 1$)

h : phase shift (+ goes left, - goes right)

k : vertical shift (+ goes up, - goes down)

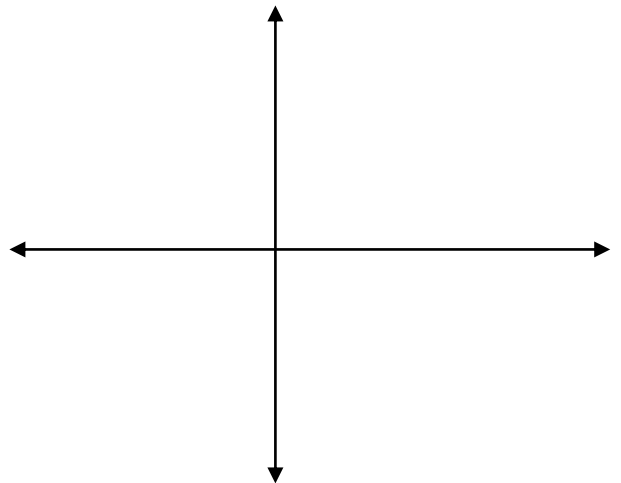
$$\text{period} = \frac{\pi}{b}$$

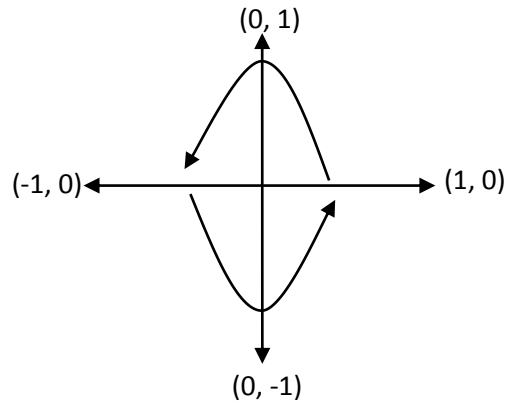
Instead of increments, we now have asymptotes (must have at least 3 asymptotes).

To find asymptotes, set $bx =$ where undefined and solve for x .

Graph and state asymptotes.

1) $y = \tan x$

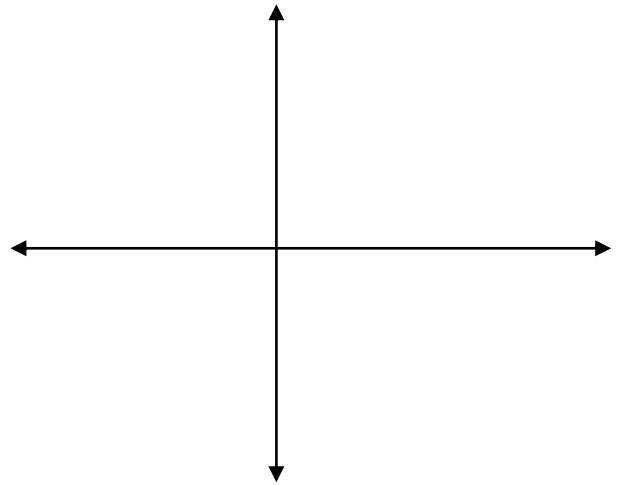




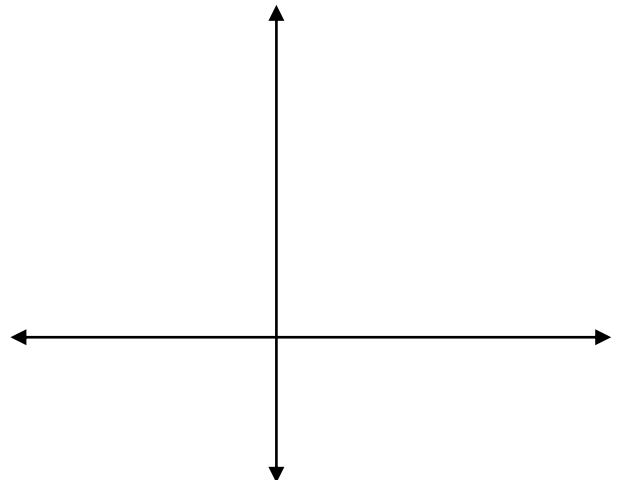
Cot is undefined at
 $-\pi, 0$ and π

Graph and state asymptotes.

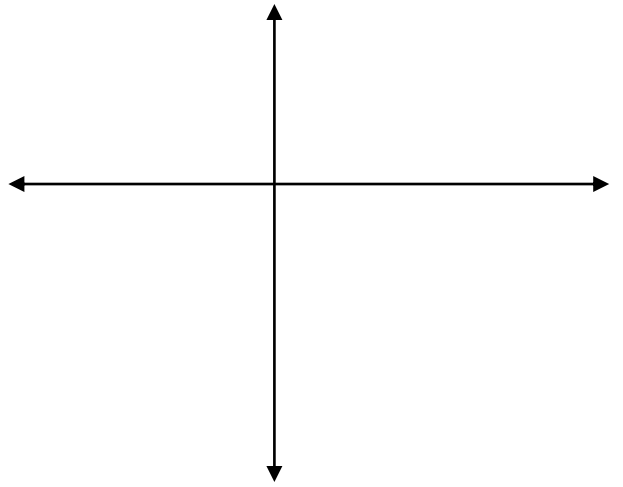
2) $y = \cot x$



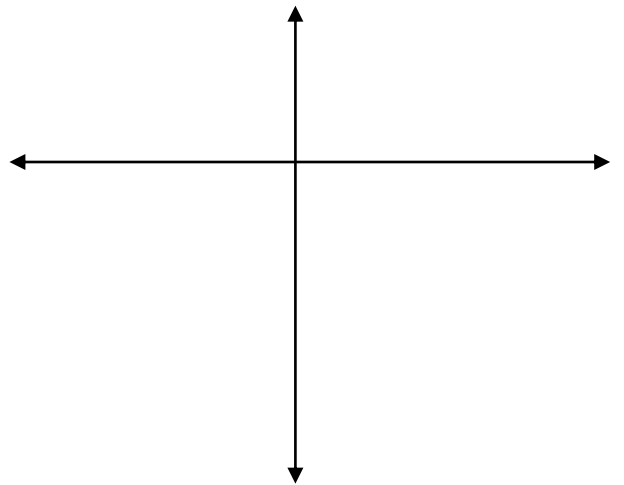
3) $y = \cot^2\left(x - \frac{\pi}{4}\right) + 3$



$$4) y = 4 \tan 3 \left(x + \frac{\pi}{6} \right) - 2$$



$$5) y = 3 \cot \left(x + \frac{\pi}{4} \right) - 4$$

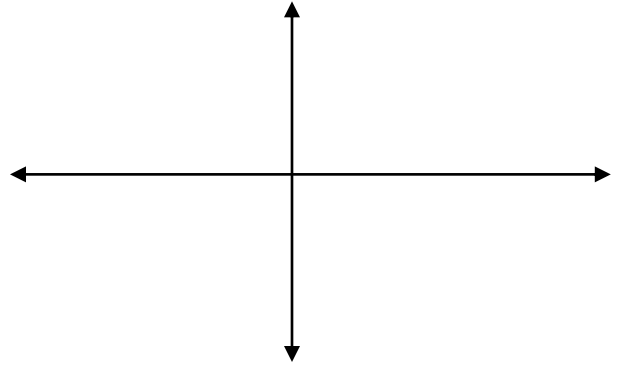


Name: _____ Date: _____ Period: _____

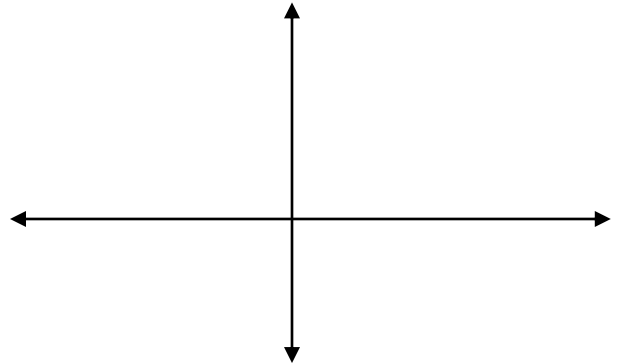
Section 4.6 Day 2: Graphing Tangent and Cotangent Functions Worksheet Day 2

Graph the following.

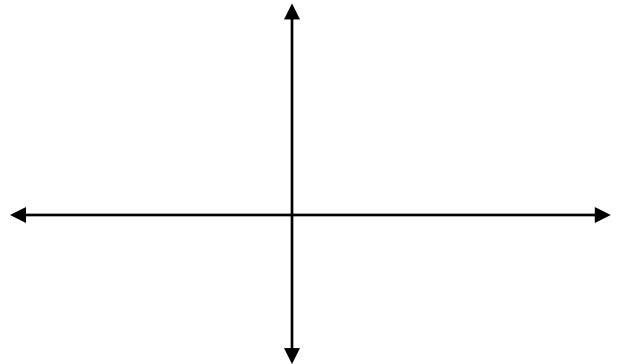
1. $y = -2 \tan(2x)$



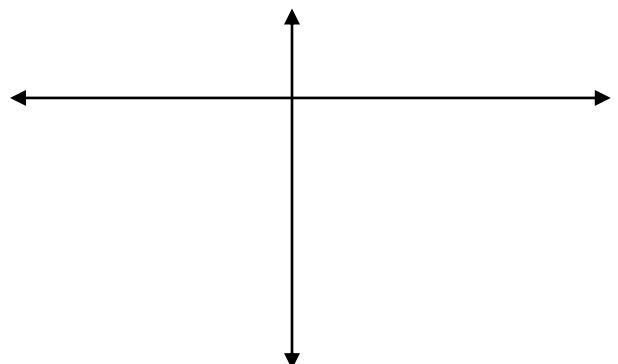
2. $y = 4 \cot(x) + 4$



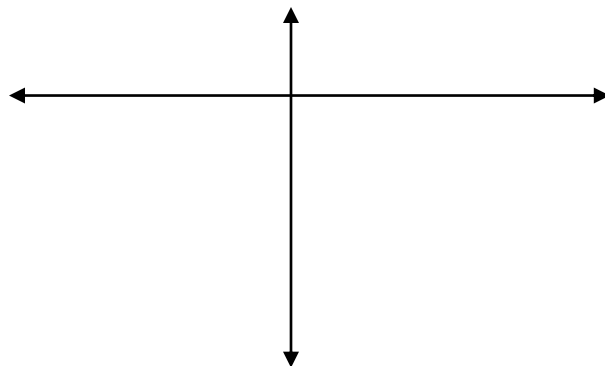
3. $y = \tan\left(2x - \frac{\pi}{3}\right) + 2$



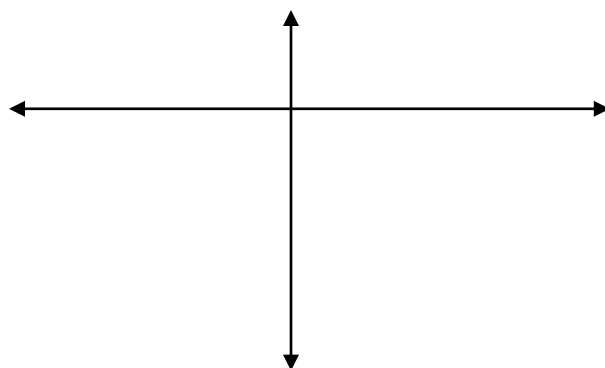
4. $y = 2 \cot\left(\frac{1}{2}x - \frac{2\pi}{3}\right) - 3$



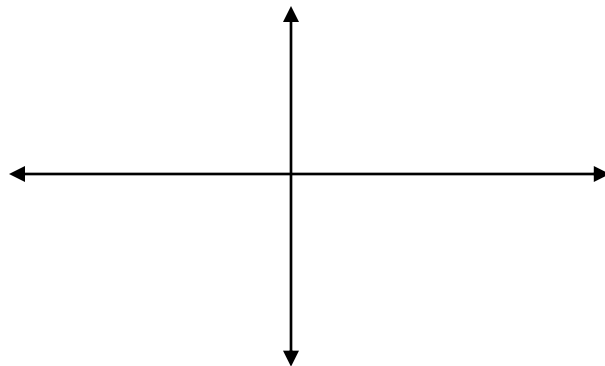
5. $y = \frac{1}{4} \cot 4x - 5$



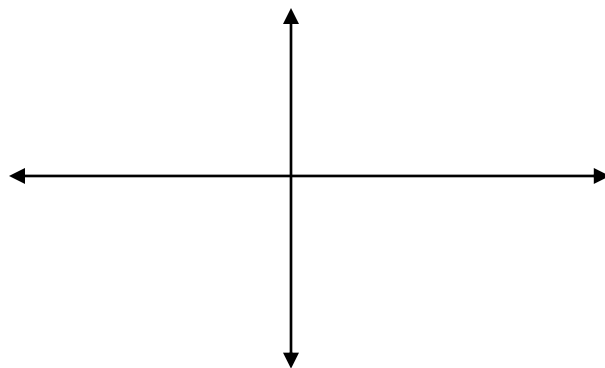
6. $y = 4 \cot \left(3x - \frac{\pi}{4} \right) - 6$



7. $y = \frac{1}{2} \tan \left(\frac{1}{2}x + \frac{\pi}{4} \right)$



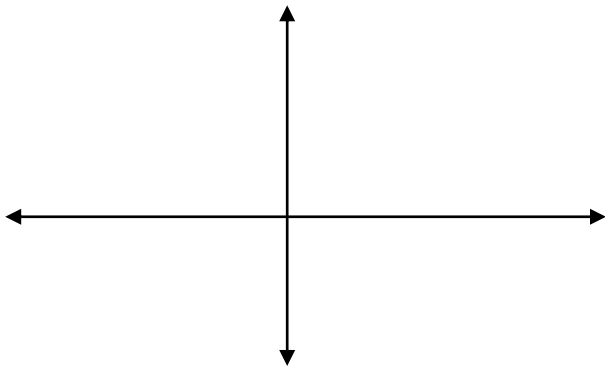
8. $y = 2 \tan \left(2x + \frac{\pi}{4} \right)$



Warm Up

Graph the following.

$$y = 2 \cot \left(2x + \frac{\pi}{3} \right) + 1$$



Name: _____

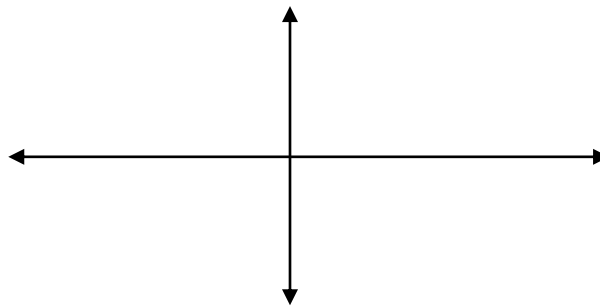
Graphing All Trig Functions Review Worksheet (Sin, Cos, Tan, Csc, Sec, & Cot)

Sketch the graph of each function for one period.

Determine the amplitude or vertical stretch, period, phase shift, and vertical shift for each.

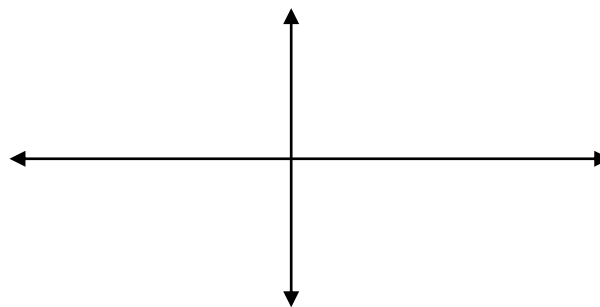
1) $y = -\sin x + 1$

Amplitude/Vertical Stretch	Period	Phase Shift	Vertical Shift



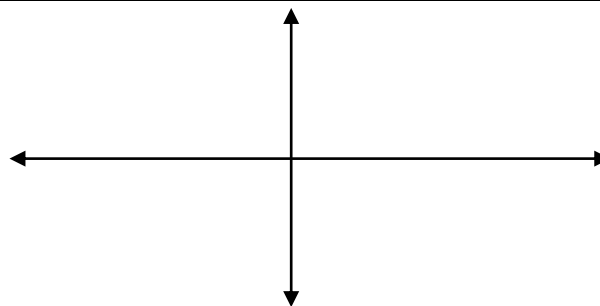
2) $y = -3 \cos 2x$

Amplitude/Vertical Stretch	Period	Phase Shift	Vertical Shift



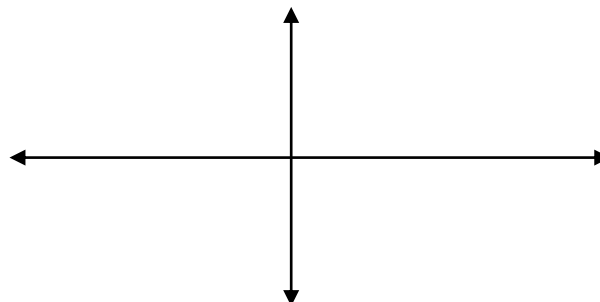
3) $y = 4 \csc\left(x + \frac{\pi}{2}\right)$

Amplitude/Vertical Stretch	Period	Phase Shift	Vertical Shift



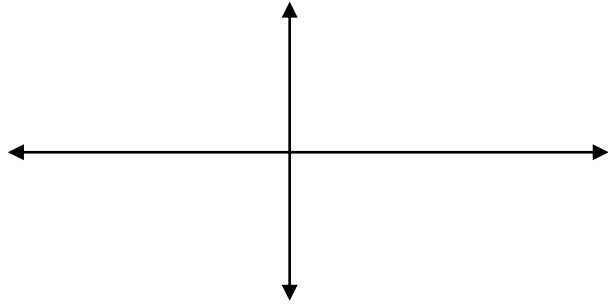
4) $y = 2 \tan 4x$

Amplitude/Vertical Stretch	Period	Phase Shift	Vertical Shift



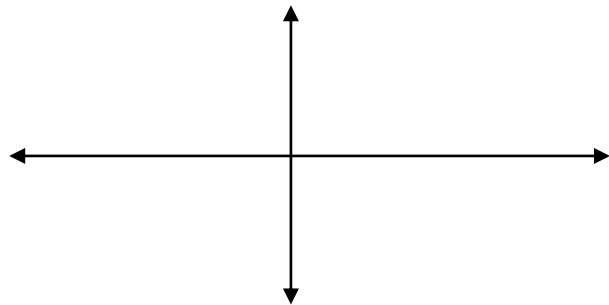
5) $y = 2\cot\left(x - \frac{\pi}{2}\right)$

Amplitude/Vertical Stretch	Period	Phase Shift	Vertical Shift



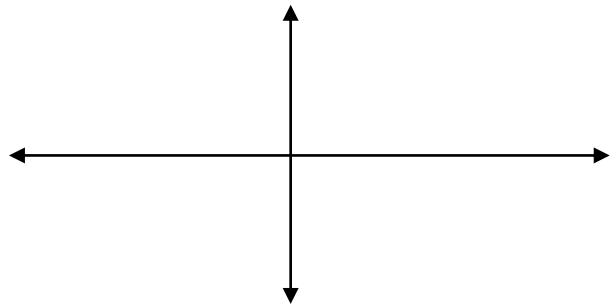
6) $y = 2\sec\left(x + \frac{\pi}{4}\right)$

Amplitude/Vertical Stretch	Period	Phase Shift	Vertical Shift



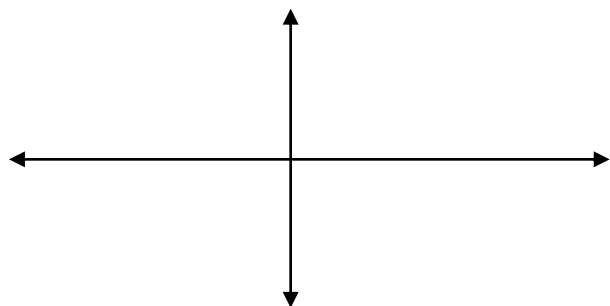
7) $y = 2\csc\left(\frac{1}{2}\left(x + \frac{\pi}{3}\right)\right) + 1$

Amplitude/Vertical Stretch	Period	Phase Shift	Vertical Shift



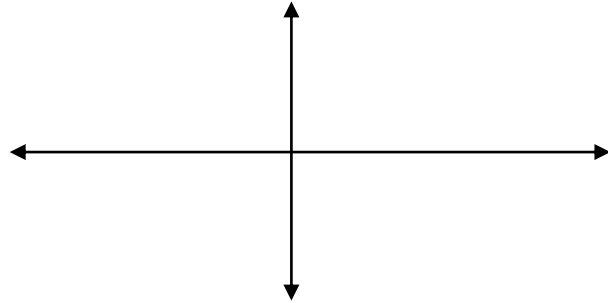
8) $y = -2\sec(2x - \pi) + 4$

Amplitude/Vertical Stretch	Period	Phase Shift	Vertical Shift



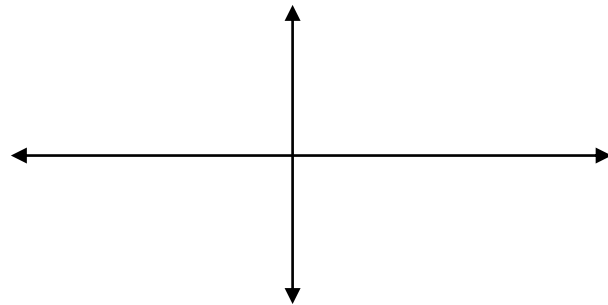
9) $y = 2 \sin\left(2x + \frac{\pi}{2}\right) + 3$

Amplitude/Vertical Stretch	Period	Phase Shift	Vertical Shift



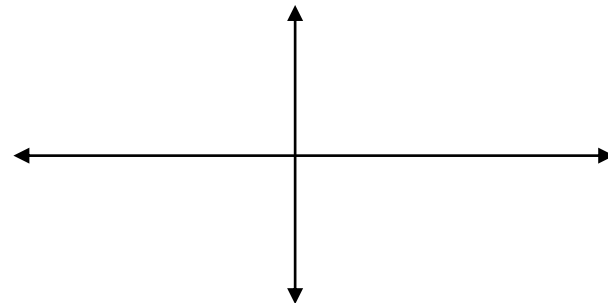
10) $y = 3 \cos(x - \pi) + 2$

Amplitude/Vertical Stretch	Period	Phase Shift	Vertical Shift



11) $y = 2 \tan\left(\frac{1}{2}x - \frac{\pi}{2}\right) - 3$

Amplitude/Vertical Stretch	Period	Phase Shift	Vertical Shift



12) $y = 2 \cot\left(x + \frac{\pi}{4}\right) + 3$

Amplitude/Vertical Stretch	Period	Phase Shift	Vertical Shift

