Math 3 – 316: Condensing Logarithmic Expressions Name: ___________________________

Yesterday, we expanded logarithmic expressions so that we could separate the inputs and look at each individually. Today, we will do the opposite, condensing multiple logarithms into a single logarithm. The primary purpose for condensing will be seen next week when we start solving logarithmic equations.

Condensing involves the same three properties we used yesterday, but it is helpful to think of them "written backwards" in a way. **NOTE: The bases must be the same!**

$$\log_a(b)+\log_a(c)=\log_a(bc)$$
$$\log_a(b)-\log_a(c)=\log_a\left(\frac{b}{c}\right)$$
$$c\log_a(b)=\log_a(b^c)$$

You may remember yesterday I recommended that you apply the properties in a specific order when expanding. That order was QUOTIENT–PRODUCT–POWER. For condensing, the order should be exactly the opposite: POWER–PRODUCT–QUOTIENT. Remember that not all these steps may apply in every case. Remember…

Property Order for Simplificiation
Expanding: Quotient -> Product -> Power
Condensing: Power -> Product -> Quotient

Example. Express as a single logarithm: $\frac{1}{3}\log(x-1)-\log(y)-4\log(z)$

$=\log[(x-1)^{1/3}]-\log(y)-\log(z^4)$ Power property

$=\log\left(\frac{(x-1)^{1/3}}{yz^4}\right)$ Quotient property

Notice since both the log(y) and log(z^4) are negative, the quotient property can be applied all at once to finish the problem. Also, there are not multiple positive logarithmic terms, so the product property does not apply.

Write the expression as one logarithm.

1. $\log_3 x+\log_3(5y)$
2. $\log_3(2z)-\log_3(x)$
3. $\log_4(3z)+\log_4 x$
4. $5\log_3 y$
5. $\log_4 x-\log_4(7y)$
6. $2\log_a x+\frac{1}{3}\log_a(x-2)-5\log_a(2x+3)$
7. $\frac{1}{3}\log_4 w$
8. $5\log_a x-\frac{1}{2}\log_a(3x-4)-3\log_a(5x+1)$
9. $\ln(y^3)+\frac{1}{3}\ln(x^3y^6)-5\ln(y)$
10. $2\log\frac{y^3}{x}-3\log(y)+\frac{1}{2}\log(x^4y^2)$
11. $2\ln x-4\ln(y)-3\ln(xy)$