Math 3 – 315: Expanding Logarithmic Expressions Name: ____________________________

Now that we have a good understanding of what logarithms are, this week we are going to focus on some properties of logarithms. You've already learned two important properties:

$$\log_a(1) = 0$$

$$\log_a(a) = 1$$

There are three other properties that are useful for simplifying logarithmic expressions. They are…

Property #1 (Product): $\log_a(bc) = \log_a(b) + \log_a(c)$

Whenever the logarithmic input is the product of multiple terms, the expression can be written as the sum of logarithms.

Property #2 (Quotient): $\log_a(\frac{b}{c}) = \log_a(b) - \log_a(c)$

Whenever the logarithmic input is the quotient of multiple terms, the expression can be written using subtraction. Notice that terms in the numerator are written as a positive logarithm, and terms in the denominator are written as a negative logarithm. This can be explained using the third property.

Property #3 (Power): $\log_a(b^c) = c\log_a(b)$

Exponents of inputs can be written as a coefficient of the logarithm. I told you the Quotient property can be explained by the Power property. Follow this reasoning:

$$\log_a(\tfrac{b}{c})$$

$= \log_a(bc^{-1})$	Exponent property
$= \log_a(b) + \log_a(c^{-1})$	Product property
$= \log_a(b) + (-1)\log_a(c)$	Power property
$= \log_a(b) - \log_a(c)$	

Let's talk about the order in which these properties are applied. Today, we are going to be focusing on **expanding** logarithmic expressions. This is when you take a logarithmic expression and write it as the sum or difference of logarithms with a simpler input. This process is primarily useful for understanding how the different terms affect your expression. Technically, as long as you apply the properties correctly, you can do them in any order; however, **when expanding**, I find this order easiest: **QUOTIENT–PRODUCT–POWER.** For example:

Expand and simplify $y = \log_3(81x)$.

$= \log_3(81) + \log_3(x)$	Product property
$= \log_3(3^4) + \log_3(x)$	$81 = 3^4$
$= 4\log_3(3) + \log_3(x)$	Power property
$= 4 + \log_3(x)$	$\log_3(3) = 1$

Expanding in this case allowed us to see that the coefficient of 81 is really just shifting the graph of $y = \log_3(x)$ up 4 units.

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Here's one more example to help jump start you on the practice set:

Express $\log_a \frac{x^3\sqrt{y}}{z^2}$ in terms of logarithms of *x*, *y*, and *z*.

$=\log_a(x^3\sqrt{y})-\log_a(z^2)$	Quotient property
$=\log_a(x^3)+\log_a(\sqrt{y})-\log_a(z^2)$	Product property
$=\log_a(x^3)+\log_a(y^{1/2})-\log_a(z^2)$	Exponent property
$=3\log_a(x)+\frac{1}{2}\log_a(y)-2\log_a(z)$	Power property

Express the following logarithms as the sum or difference of logs with simplest inputs possible.

1. $\log_4(xz)$
2. $\log_4(\frac{y}{x})$
3. $\log_4(\sqrt[3]{z})$
4. $\ln(\frac{xz}{y})$
5. $\log_3(xyz)$
6. $\log_3(\sqrt[5]{y})$
7. $\log_a(\frac{x^3w}{y^2z^4})$
8. $\ln(\frac{y^5w^2}{x^4z^3})$
9. $\log\left(\frac{\sqrt[3]{z}}{x\sqrt{y}}\right)$
10. $\log\left(\frac{\sqrt{y}}{x^4\cdot\sqrt[3]{z}}\right)$
11. $\ln(x^2+5x+6)$
12. $\ln(5x+10)$

Expanding can also be useful if you know some simpler logarithmic values because you can use this tool to evaluate bigger logarithms. For example, if you know that $\log(2)=0.301$, you can figure out a bigger logarithm like $\log(800)$ ...

$=\log(8\cdot100)$	
$=\log(8)+\log(100)$	Product property
$=\log(2^3)+\log(10^2)$	$8 = 2^3$; $100 = 10^2$
$=3\log(2)+2\log(10)$	Power property
$=3\cdot0.301+2\cdot1$	Substitution
$=2.903$	Arithmetic

Given $\log(2)=0.301$, $\log(5)=0.699$, and $\log(7)=0.845$, evaluate the following logarithms without a calculator.

13. $\log(70)$
14. $\log(140)$
15. $\log(500)$
16. $\log(175)$
17. $\log(250)$
18. $\log(320)$
19. $\log(686)$
20. $\log(3500)$
21. $\log(350)$
22. $\log(1400)$
23. $\log(280)$
24. $\log(245)$