Math 3 – 310: *e* (Euler's Number) Name: ____________________

A common application of exponential functions is financial formulas. Let's look at a rather simple example that you most likely will gain experience with at some point if you haven't already. When you deposit money into a savings account, your money doesn't just sit in the vault. The bank (or other financial institution) actually invests your money so that they can make money themselves. In exchange for using your money, they will periodically deposit some extra money into your account. Basically, they are paying you for letting them use your money. This payment is called **interest**, and is typically expressed as a percentage of the money you have deposited in your account. When the bank deposits interest annually (once per year), you have earned **simple interest**. What you actually want is for the bank to deposit interest into your account more frequently; this is called **compound interest**. You may have seen the formula for compound interest situations before: $A = P(1+\frac{r}{n})^{nt}$. In this formula, A is the amount of money in your account after t years. P is the initial amount (aka the "principle amount"), and r is the <u>annual</u> interest rate expressed as a decimal. n is the number of times the money is compounded (meaning, interest is deposited into your account) every year. Let's look at an example:

When I was born, my great-grandfather deposited $5,000 into a savings account that paid 6% interest compounded monthly. When I turned 18, I became the owner of the account. Let's see how much money I just inherited to pay for college.
In this case, P = $5000, r = 0.06, and t = 18. These are explicitly stated in the problem. The problem also says the interest is compounded monthly, meaning 12 times per year, so n = 12. Using the formula,

$$5000(1+\frac{.06}{12})^{12\cdot 18} = 5000(1.005)^{216} = \$14683.83$$

Let's investigate some of the pieces to the formula and what they are doing. $\frac{r}{n}$ is added to 1 to form the base. Remember this is in decimal form; you know that 1 is the decimal form of 100%. So the base of the formula takes into account 100% of the money that is already in the account, and then adds an additional $\frac{r}{n}$ each compounding period. nt calculates how many compounding periods have occurred over t years.

Now it is your turn. This example is designed so that you see the impact of compounding your money as much as possible.

1. You open a bank account by depositing $1. This account pays 100% interest annually. Complete the table for the various values of n to determine how much money you will have at the end of 1 year. Round to 4 decimal places if necessary.

n	A	n	A
1		52 (weekly)	
2		365 (daily)	
4 (quarterly)		8760 (hourly)	
6		525600 (every min)	
12 (monthly)			

2. What trends or patterns do you notice about the impact of n on the compound interest formula?

Math 3 – 310: *e* (Euler's Number) Name: ____________________________

That whole exercise, while good for you to understand compound interest, was actually to introduce a very important mathematical constant (kind of like π). This one is the number *e*. It is called Euler's number, after a very famous mathematician named Leonhard Euler. One way to define *e* is to look at the limit of the function that you were just investigating: $(1+\frac{1}{n})^n$. In your table, you noticed that eventually you reached a point where increasing *n* did not have any noticeable change on the amount of money in your account. If we had continued increasing *n* to infinity, essentially saying that your bank account is compounded **continuously**, we would find the value of *e* to be 2.718281828459045… *e* is actually irrational, meaning its decimal form does not end or repeat.

Now that we have met *e*, let's have a little Q&A session so we can get to know this number better.

3. When will we use *e*?

   e will primarily be used in situations where something experiences continuous exponential growth or decay. Examples include an account that is compounded continuously, populations (human, bacterial, etc.) could be modeled using continuous growth, and others we will get to when we talk about word problems later.

4. Is *e* related to any special notations or functions?

   Actually, yes. When *e* was first defined by John Napier in the early 17th century, he was examining exponential functions. *e* is the base that causes an exponential function to have a slope of 1 at the y-intercept (0, 1). Napier's work was later adopted into work by other mathematicians, and eventually the **natural logarithm**, written in function notation as ln(*x*), was born. Mathematically, $\ln(x)=\log_e(x)$. It is called the "natural" logarithm because *e* is a number that naturally appears in many applications of mathematics, including calculus and series. Since it is a logarithm, all of the properties that we learn and talk about for log(*x*) also apply to any expressions or equations written in terms of ln(*x*).