Math 3 – 303: Rational Exponents Name: ___________________________

All of the properties that we learned on 301 apply to every exponential expression, but so far we have only simplified expressions with integer exponents. It is possible to have rational exponents (fractions), and you've actually worked with them before but in disguise.

Rational exponents can also be written in radical form.

Property #8 – Rational exponents: $x^{a/n} = \sqrt[n]{x^a}$

In the radical notation $\sqrt[n]{x^a}$, *n* is called the **index** of the root. The expression $\sqrt[n]{x^a}$ would be read "the *n*-th root of *x* to the *a*-th power." For square roots, which you have experience using, *n* = 2 and is usually not written outside the radical because it is the most basic case. You would not use *n* = 1 because that is the same as having an integer exponent. So here is an example of how you have been using rational exponents without knowing:

$$\sqrt{25} = (25)^{1/2} = (5^2)^{1/2} = 5$$

Notice in the first step, $\sqrt{25}$, there is no value written for *n*, so this is the square root of 25. Using Property #8, we can rewrite this radical into exponential form… using a fraction for an exponent. Step 3, $(25)^{1/2} = (5^2)^{1/2}$, is the key step because here we can see the transition to the properties that we learned yesterday. **This is the second "new" idea for today: certain integers can be rewritten as exponential expressions in order to aid the simplification process. This is called "changing the base."** It will be especially useful once we start solving exponential equations.

Determine the value of the following exponential expressions without using a calculator. (Hint: Change the base so that it is an exponential expression and the denominator of the exponent is eliminated when the "Power of a power" property is applied.)

1. $49^{1/2}$
2. $32^{3/5}$
3. $8^{5/3}$
4. $27^{2/3}$
5. $16^{3/4}$
6. $64^{2/3}$
7. $1000^{4/3}$
8. $81^{1/4}$

We can also use this relationship between radical and exponential expressions to make it easier to simplify radical expressions. We already know all of the exponential properties necessary to simplify $\sqrt[3]{\dfrac{8x^6y^{12}}{x^3}}$, as long as we can translate the radical expression into an exponential one.

Remember, using Property #8, that $\sqrt[3]{\dfrac{8x^6y^{12}}{x^3}} = \left(\dfrac{8x^6y^{12}}{x^3}\right)^{1/3}$, and you did questions like this for homework last night. Simplify the radical expressions below:

9. $\sqrt[3]{\dfrac{8x^6y^{12}}{x^3}}$
10. $\sqrt[3]{\dfrac{a^6b^{-5}}{27a^{-3}b}}$
11. $\sqrt[4]{\dfrac{5x^{-2}y^{12}}{x^7}} \cdot \sqrt[4]{\dfrac{125y^{-3}}{x^3y}}$

There is one way these simplification problems can be more complicated. In all of the previous questions, the exponents eventually worked out to be integers. This is not always the case. To simplify rational exponents as a radical expression, you will need to determine the quotient and remainder of the exponent. For example, $x^{7/4} = x\sqrt[4]{x^3}$. Look at the initial exponent: $7/4$. If you divide 7 by 4, the quotient will be 1 because there is one full group of 4 in a group of 7. That is why there is one *x* outside the radical sign when it is simplified. Also when you divide 7 by 4, the remainder is 3 because there are 3 left over… they cannot make a full group of 4. That is why there are 3 *x*'s left inside the radical. Essentially, the intermediate step looks like this: $x^{7/4} = x^{4/4} \cdot x^{3/4} = x\sqrt[4]{x^3}$. Notice the middle step is like a reverse application of the Product of Like Bases Property Try writing the following exponential expressions as simplified radicals.

12. $x^{10/3}$

13. $y^{8/3}$

14. $z^{9/2}$

15. $b^{21/4}$