Math 3 – 301: Review of Exponential Properties Name: ____________________________

We've spent the last month discussing polynomial functions, in which the variable was raised to an exponent. In the next unit, we will start by reviewing exponential functions, where the variable **is** the exponent. Our primary goal that we will reach in a few weeks is to investigate the relationship between exponential functions and a new type of function: logarithms.

Our starting point will be to review properties of exponents. You have worked with these before in both Math 1 and 2. At the start, we must establish our vocabulary… the **base** is the constant or variable being multiplied, and the **exponent** is the number of times the base is multiplied. Exponential expressions are written b^x, where *b* is the base and *x* is the exponent.

Property #1 – Product of like bases: $b^m b^n = b^{m+n}$

The key here is that the bases must be the same. This property should come as no surprise to you. You already know this. For example,

$$2^4 \cdot 2^3 = (2 \cdot 2 \cdot 2 \cdot 2) \cdot (2 \cdot 2 \cdot 2) = 2^7$$

Remember that exponents are just counting the number of times the base is multiplied, so as long as the bases are the same, the exponents can be added together to combine the two terms. Now we just need to apply it in general. Simplify the following expressions into a single exponential term.

1. $b^2 \cdot b^9$
2. $c^x \cdot c^{4x}$
3. $d^y \cdot d^3$
4. $k^x \cdot k^{-2}$

Property #2 – Quotient of like bases: $\frac{b^m}{b^n} = b^{m-n}$

Here's another one that you already know. You've been cancelling factors since middle school. For example, you know that

$$\frac{2^7}{2^3} = \frac{2 \cdot 2 \cdot 2 \cdot 2 \cdot \not{2} \cdot \not{2} \cdot \not{2}}{\not{2} \cdot \not{2} \cdot \not{2}} = 2^4$$

Subtracting the exponents tells us the difference in the number of times the base is being multiplied, meaning those terms will be left over after we cancel. Use this property to simplify the following expressions into a single exponential term.

5. $\frac{b^9}{b^2}$
6. $\frac{c^{4x}}{c^x}$
7. $\frac{d^y}{d^3}$
8. $\frac{k^x}{k^{-2}}$

Property #3 – Power of a power: $(b^m)^n = b^{mn}$

This property is also common sense if you stop to think about it.

$$(2^4)^3 = 2^4 \cdot 2^4 \cdot 2^4 = (2 \cdot 2 \cdot 2 \cdot 2) \cdot (2 \cdot 2 \cdot 2 \cdot 2) \cdot (2 \cdot 2 \cdot 2 \cdot 2) = 2^{12}$$

You might remember from allll the way back in elementary school that multiplication was first presented to you as a quicker way of doing addition. Instead of adding 6 seven times, you can just multiply $6 \cdot 7$ and you will get the same answer. Exactly what is going on in this property. Simplify...

9. $(b^2)^9$
10. $(c^x)^{4x}$
11. $(d^3)^y$
12. $(k^x)^{-2}$

Property #4 – Power of a product: $(a^m b^n)^x = a^{mx} b^{nx}$

The power of a power property extends itself into this fourth property: the power of a product. The explanation for this one is essentially the same as the explanation for Property #3. Many students mistake this for being related to the Distribution Property. While technically incorrect, connecting these two is not the worst mistake because they are similar ideas.

13. $(a^3b^2)^4$ 14. $(cd^x)^4$ 15. $(e^3f^y)^{-2}$ 16. $(g^xh^2)^{2x}$

Property #5 – Power of a quotient: $\left(\frac{a^m}{b^n}\right)^x = \frac{a^{mx}}{b^{nx}}$

This is another extension of the power of a power property; if you are having trouble understanding Property #5, we need to go back and check your understanding of Property #3.

17. $\left(\frac{a^3}{b^2}\right)^4$ 18. $\left(\frac{c}{d^x}\right)^4$ 19. $\left(\frac{e^3}{f^y}\right)^{-2}$ 20. $\left(\frac{g^x}{h^2}\right)^{2x}$

The last two properties that we cover are different, in that they are actually just definitions. It is just the way things are, no matter what.

Property #6 – Zero exponent: $b^0 = 1, b \neq 0$

Pretty sure everyone has at least heard that anything to the 0th power equals 1. Well that is almost correct. Zero to the 0th power (0^0) is actually undefined. But the statement is true for every other possible base. Another thing to be very careful about, and we have talked about this many times so it is also not new:

"Negative seven to the 0th power" is written $(-7)^0$ and it equals 1.

"The opposite of seven to the 0th power" is written -7^0 and it equals –1.

21. b^0 22. $(c^2)^0$ 23. $(7d^4)^0$ 24. $(g^xh^2)^0$

Property #7 – Negative exponent: $b^{-n} = \frac{1}{b^n}$ *or* $\frac{1}{b^{-n}} = b^n$

Notice the negative exponent converts the exponential expression into its multiplicative inverse (aka its reciprocal). Once the effect of the negative is taken into account, the exponent becomes positive.

25. b^{-9} 26. $(c^2)^{-x}$ 27. $(7d^4)^{-2}$ 28. $\frac{k^5}{k^{-2}}$