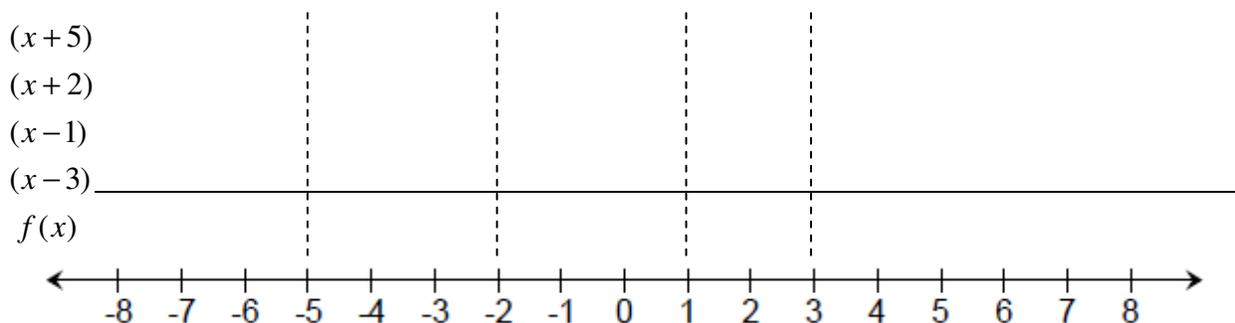


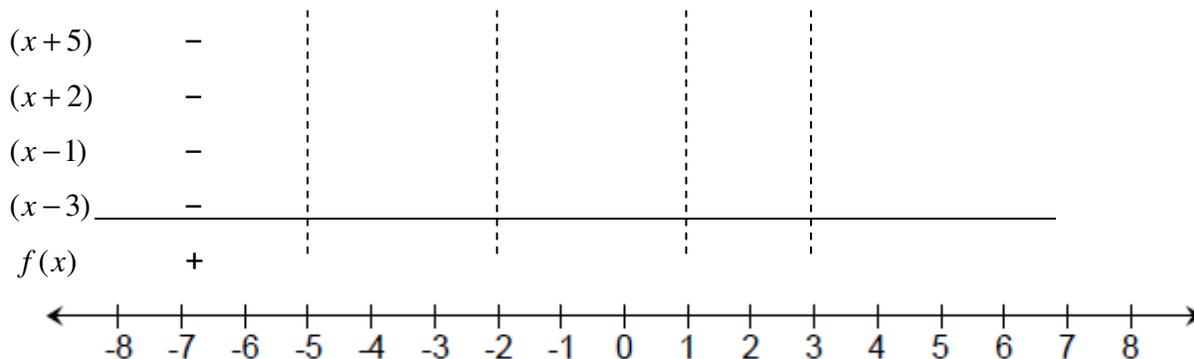
For those of you who despise sketching, there is an alternative method for solving polynomial inequalities called **sign tables**. Basically, it is an organized way of substituting values into the inequality and checking whether this makes the inequality true or false.

To start with, a sign table requires the factored form of the polynomial. You need to find the real roots first to determine what values to substitute. So let's say we factored a polynomial and ended up with the inequality $(x + 5)(x + 2)(x - 1)(x - 3) > 0$. This polynomial has roots at $x = -5, -2, 1,$ and 3 . As you know from all of your sketching experience, the roots are where the value of the function might change sign from positive to negative (or negative to positive), so they are important for solving inequalities. Let's talk about how to set up our sign table.

First, think of the base of your sign table like a number line (or the x -axis). Second, we need a row for each of the factors. Lastly, draw a vertical line at each root. So our table for this example would look something like this...



We are going to use the bottom row to tally values and determine whether the function $f(x)$ is positive or negative for each section of x -values. Each of the vertical lines divide the x -values into intervals. The main point to filling in the sign table is to **pick an x -value from each interval, plug it into each of the factors, and then determine whether your function will be positive or negative for that interval**. For example, the first interval we have is $(-\infty, -5)$, so I will use $x = -7$. As I plug -7 into each factor, I get negative numbers each time $(-2, -5, -8, -10)$. If I multiply 4 negative numbers together, I will end up with a positive result. This means that at $x = -7$, and all other x values in this particular interval, $f(x)$ is positive, or >0 . So my sign table now looks like this.



Finish the rest of the table by selecting values for x in each of the other intervals and following the same procedure.

The final part of solving the inequality is determining which intervals are part of our solution. Since the inequality we are considering is > 0 , we want to find all of the intervals where $f(x)$ is positive. There should be three: $(-\infty, -5)$, $(-2, 1)$, and $(3, \infty)$. Since the inequality sign is $>$, we will be using parentheses to write the final solution in interval notation: $(-\infty, -5) \cup (-2, 1) \cup (3, \infty)$.

To summarize, here are the steps you will need to think through to properly analyze a polynomial inequality using a sign table.

(1) *Factor*

- Set inequality to zero
- Divide out any GCF
- Choose factoring method(s)
- Determine **real** zeros (just like solving equations)

(2) *Sign Table*

- Construct with one row for each factor and vertical lines representing each root
- Select a value for x from the first interval
- Substitute into each factor and determine whether it is positive or negative
- “Multiply” the signs together to determine whether the function is positive or negative
- Repeat for each

(3) *Use interval notation to write solution*

- Determine which intervals satisfy the inequality.
- Write the solution

Use sign charts to solve the following inequalities.

1. $(x + 3)(x - 2)(x - 4) < 0$

2. $(-x)(2x + 3)(x - 1) \geq 0$

3. $(x + 3)(x - 3)(2x - 1) \leq 0$

4. $(x + 3)(x^2 - 3x + 9) < 0$

5. $x(x + 3)(x - 2)(x - 4) > 0$

6. $(5x + 2)(7x - 2)(x - 4) \geq 0$

7. $x^3 + 3x^2 - 4x - 12 \leq 0$

8. $x^4 - 5x^2 > 6$

9. $x^4 - 4x^3 - x^2 + 4x \geq 0$

10. $x^3 - 4x^2 - 4x + 16 \leq 0$

11. $x^3 + 3x^2 > x$

12. $2x^3 - x^2 \leq 2x - 1$

13. $-x^4 + 9x^2 + 22 \geq 0$

14. $5x^4 > 405$

15. $15x^3 + 10x^2 - 25x \leq 0$

16. $x^5 + x^4 - 3x^3 - 3x^2 - 4x - 4 > 0$