This week we are going to change gears slightly. In the previous two weeks, we have been solving equations. Now we will learn how to solve inequalities. There is a slight conceptual difference between solving equations and solving inequalities. Let's use a rocket as an example, and assume you have a polynomial function that tells you the rocket's planned altitude any time after the launch. If you were solving an equation, you could tell when a rocket reaches a certain altitude. Solving an inequality could tell you the full interval of time when the rocket was above or below that certain altitude. So sometimes it is more helpful to solve an inequality.

There are two main ways to solve inequalities: graphically and analytically. Today we are going to focus on solving by sketching a graph because it uses two skills we already know. Here are the basic steps to solve a polynomial inequality by sketching.

(1) *Factor*
- Set inequality to zero
- Divide out any GCF
- Choose factoring method
- Determine **real** zeros (just like solving equations)

(2) *Sketch*
- Determine end behavior from degree and leading coefficient
- Place real zeros on x-axis
- Check multiplicities at zeros

(3) *Use interval notation to write solution*

<u>Interval Notation</u>

Interval notation is a simple way of writing all of the numbers between two other numbers (the boundaries). For example, if you want to write that your solution is all of the numbers between 2 and 5, you would write (2, 5). There are a couple things to consider when using interval notation.

(1) ( ) or [ ]?
Parentheses ( ) are used when a boundary is not included in the solution. Typically, this is when the inequality sign is $<$ or $>$. It is also important to use parentheses if infinity is a boundary because you cannot include infinity. Square brackets [ ] are used when the boundary is part of the solution. This usually is if the inequality sign is $\leq$ or $\geq$.

(2) What if I have multiple intervals that are solutions?
This happens pretty frequently. If you have two or more intervals that satisfy the inequality, we need a way to include both of them in the interval notation. This is done with the symbol $\cup$, which stands for "union." You are joining (or uniting) the interval notations into the same solution statement.

For example, let's say we want to know when a function is ≥ 0, and the sketch is above zero between $x = 2$ and $x = 5$, and then again starting at $x = 7$ and going to ∞. The intervals $[2, 5]$ and $[7, \infty)$ are both part of the solution, so the interval notation statement would be $[2, 5] \cup [7, \infty)$.

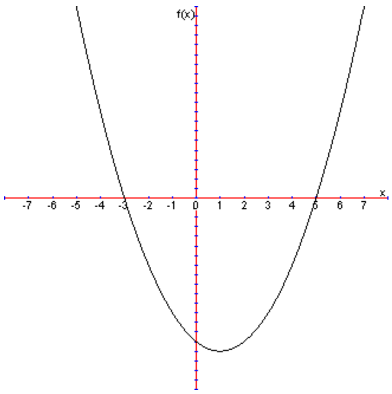
Solve the inequalities.

Example

1. $x^2 - 2x - 15 > 0$

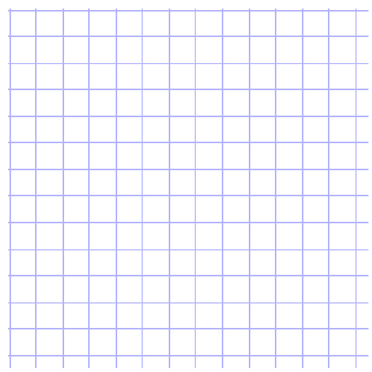
$(x-5)(x+3) = 0$

$x = 5, \ x = -3$



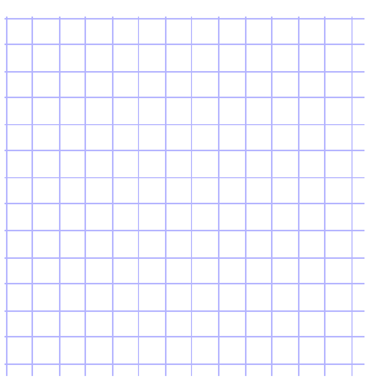
I want to know when the function is > 0,
so when is it above the x-axis?
From **–∞** to –3, and then again from 5 to ∞.
Solution: (–∞, –3)∪(5, ∞)

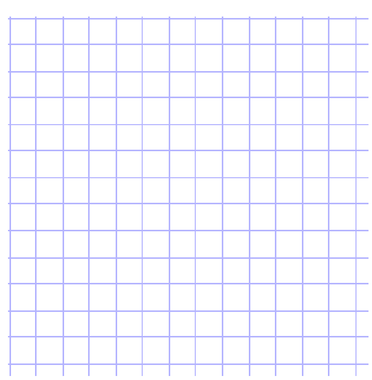
2. $x^2 + 6x + 8 \le 0$



3. $x^2 - 4x - 12 \ge 0$

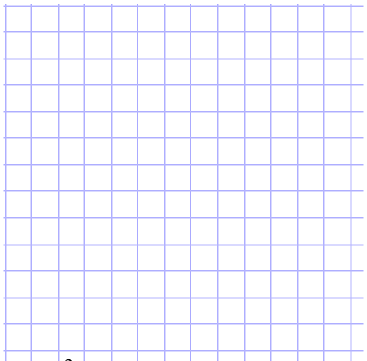
4. $x^2 - 8x + 12 < 0$

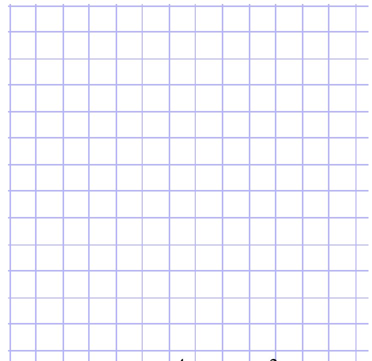




5. $x^3 + 5x^2 - 4x - 20 > 0$

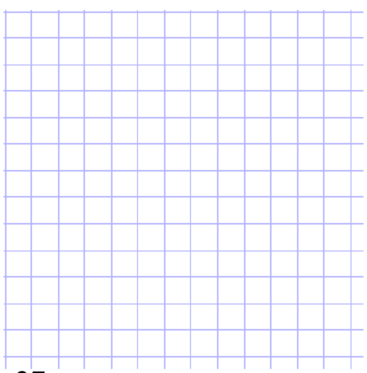
6. $6x^3 - 15x^2 - 4x \le -10$

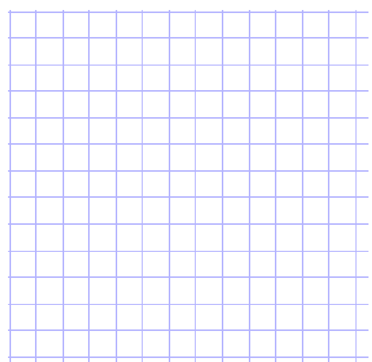




7. $7x^3 + 15x^2 + 2x \ge 0$

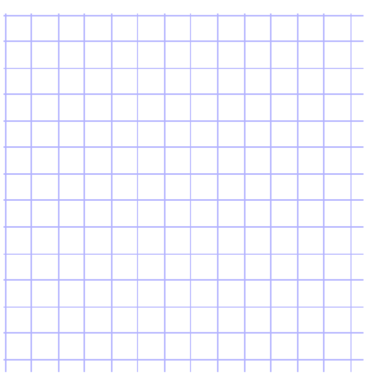
8. $x^4 - 13x^2 + 30 < 0$

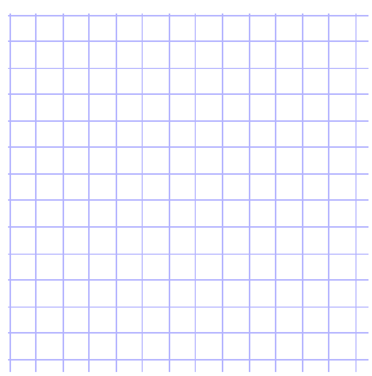




9. $8x^3 > 27$

10. $6x^4 - 23x^2 \ge -7$





11. $x^3 + x^2 - 36x > 36$

12. $x^4 + 2x^3 - 27x \geq 54$

13. $x^5 - 10x^3 + 21x > 0$

14. $x^4 - 2x^3 - 8x + 16 < 0$

15. $4x^5 + 16x^4 - 11x^3 - 44x^2 - 20x \leq 80$

16. $5x^2 - 3x \geq 2$