CCGPS Adv Alg – 211: Graphing Higher-Order Polynomials Name: ______________________________

So far in this unit, we have talked about a variety of characteristics of polynomial functions: degree, leading coefficient, end behavior, y-intercept, and roots. You learned to identify the first four from a function written in standard form, like $f(x) = x^4 - x^3 - 7x^2 + x + 6$. Now we are going to adapt our understanding of these characteristics so that we can identify them from a function written in factored form, like $f(x) = (x+2)(x+1)(x-1)(x-3)$.

Degree/Leading Coefficient:
Recall from previous discussions that the degree of a polynomial is equivalent to the largest exponent, so $f(x) = x^4 - x^3 - 7x^2 + x + 6$ is a 4th degree polynomial. We learned Monday that it is a little more difficult to identify the degree in factored form, but not much. Look at $f(x) = (x+2)(x+1)(x-1)(x-3)$. One way (the long way) to identify the degree would be to multiply all of those factors together and combine "like terms" until it is fully simplified. The shorter way is similar, but does not require you to multiply everything out, just the *x* terms. Hopefully you recognize that the highest degree term possible will be the product of all of the *x* terms, so just look at those. In our case, the product of *x* and *x* and *x* and *x* will be x^4.

This method will also allow us to identify the leading coefficient. A slightly more complicated function, like $g(x) = (2x+3)(x+1)(3x-1)^2$ will be a good example to explore. Remember, **the highest degree term will be the product of all the *x* terms.** So, our highest degree term will be

$$2x \cdot x \cdot 3x \cdot 3x = 18x^4$$

Don't forget the factor (3*x* – 1) has a multiplicity of 2, so that is why the 3*x* is included twice. From our product, we have learned that the degree is 4 and leading coefficient is 18.

It is important to find the degree and leading coefficient of your polynomial because, as you learned previously, they work together to determine the **end behavior** of the function. Knowing the end behavior is an important part of correctly sketching the function.

Y-intercept:
Good news! You already know how to find the y-intercept, because you know that for any kind of function, the y-intercept occurs where ____ equals zero. So determining the y-intercept is still just as easy as substitution. For example, the y-intercept of $f(x) = (x+2)(x+1)(x-1)(x-3)$ is

$$f(0) = (0+2)(0+1)(0-1)(0-3) = 6$$

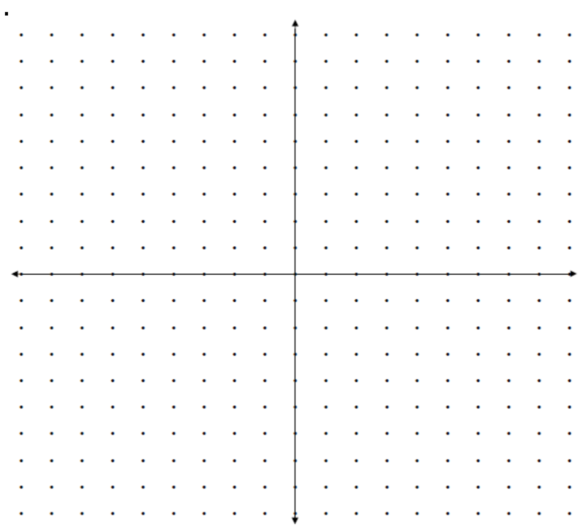
Putting it all together…

(1) Sketch the function $g(x) = (2x+3)(x+1)(3x-1)^2$.

(a) Degree and leading coefficient →End behavior:

(b) Y-intercept:

(c) Zeros:



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On a separate sheet of graph paper, sketch the following functions:

(2) $a(x) = (x-1)^2(x-5)^3$

(3) $b(x) = (x+2)^4(x-1)^3(x-4)$

(4) $c(x) = x^2(x+1)$

(5) $d(x) = (x+6)^2(3x-2)^2$

(6) $e(x) = x(4x+3)(x-8)^2$

(7) $f(x) = x^2(x+4)(2x-3)^3$

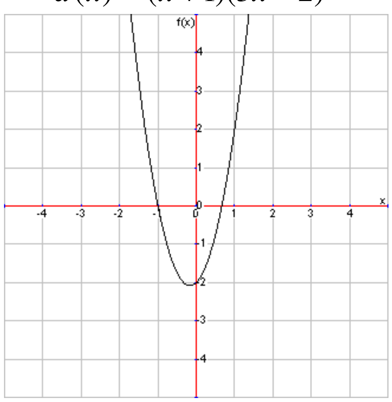
(8) $g(x) = (x+7)^2(x+4)^2(x+2)^3$

(9) $h(x) = x(x+1)(x-1)^2(x-3)^2$

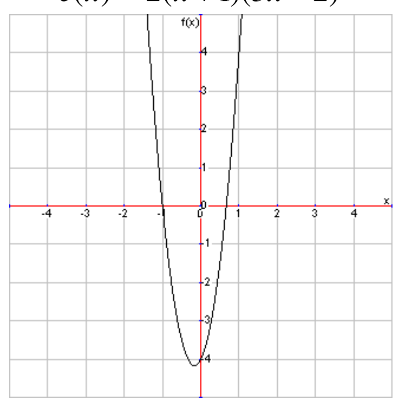
Scale Factors:
Occasionally, your function will include a constant multiplied with the factors. This is called the **scale factor**. In the example function $f(x) = 5(x+2)(x+1)(x-1)(x-3)$, the 5 is the scale factor.

The scale factor affects two things: the _ _ _ _ _ _ _ _   _ _ _ _ _ _ _ _ _ _ _ _ _ and the __ - _ _ _ _ _ _ _ _ _ _ _ of the function. Let's see what happens graphically by comparing $d(x) = (x+1)(3x-2)$, which does not have a scale factor, to $e(x) = 2(x+1)(3x-2)$, which has a scale factor of 2.

$d(x) = (x+1)(3x-2)$



$e(x) = 2(x+1)(3x-2)$



(10) Did the location of the zeros change? Did the y-intercept change? Did the shape of the graph change?

On a separate sheet of graph paper, sketch the following functions:

(11) $j(x) = (2x+1)(x-2)^2(x-5)$

(12) $k(x) = (x+2)^2(3x-1)^3(x-4)^2$

(13) $m(x) = 3x^2(x+1)$

(14) $n(x) = (3x+4)^2(x-2)^3$

(15) $p(x) = 2x^3(x+3)(x-4)^2$

(16) $q(x) = 3x^2(x+4)^2(x-3)^3$

(17) $r(x) = -(x+2)^2(x-4)^2(x-2)^3$

(18) $s(x) = -2x(x+3)^3(x-1)^2(x-3)$

<u>Other Disturbing Wrinkles</u>:
There are a few other little wrinkles that can be added to these problems, but they should not throw you off too badly. Let's look at some of them.

Factors with negative slopes
So far, we have not discussed or practiced problems where a factor has a negative slope, like $f(x) = (-2x-1)(x+4)(2x-3)^2$. Here, the first factor listed is $(-2x-1)$; the coefficient of the *x* term is –2. NOTHING CHANGES!

Determine the degree and leading coefficient exactly the same way: $(-2x)(x)(2x)(2x) = -8x^4$
Determine the y-intercept by substituting 0 for *x*: $(-2\cdot 0-1)(0+4)(2\cdot 0-3)^2 = -36$
Determine the zeros by solving for the values that make each factor equal zero:

$$-2x-1=0 \rightarrow -2x=1 \rightarrow x=-0.5$$

Nonlinear factors
We also have not discussed nonlinear factors. An example would be a function like $g(x) = (x-1)(x^2+2x-5)$. This function has a quadratic (2nd degree) factor, but guess what? NOTHING CHANGES (but you do have to do a little more work).

Determine the degree and leading coefficient exactly the same way: $(x)(x^2) = x^3$

> Notice here we do not need to consider the 2*x* term. This is because our goal is just to determine the highest degree term of our function. That only will come from multiplying the highest degree term of each factor. The 2*x* is 1st degree; the x^2 is 2nd degree, so it is more important than 2*x*.

Determine the y-intercept by substituting 0 for *x*: $(0-1)(0^2+2\cdot 0-5) = 5$
Determine the zeros by solving for the values that make each factor equal zero (here comes more work):

> Solving the first factor is no sweat. Anyone remember how to solve this $x^2+2x-5=0$ for *x*? It can't be factored, otherwise it wouldn't be written as a quadratic. Here's a hint:

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Nonlinear factors with imaginary solutions
These can be a real bugger, because you follow the same steps as nonlinear factors, do all the work, and nothing changes. An example is $h(x) = (x-1)(x^2+x+1)$. Following all of our previous steps, you could determine the degree and leading coefficient (and the end behavior from these) and the y-intercept, but the difference comes when determining the zeros. If you use quadratic formula to solve for the zeros of the second factor, you will find they are imaginary. **Imaginary zeros cannot be graphed on our coordinate plane; we only deal with real numbers.** So the graph of *h(x)* will only have one real zero: at *x* = 1. We don't work much with these now because this unit is focused on graphing, but it is important for you to know this exists. We will work more with these functions in Unit 4 (January).

On a separate sheet of graph paper, sketch the following functions:

(19) $t(x) = -x^2(x+3)(x-2)^3$
(20) $w(x) = (x+2)^2(3-x)(2x-9)$
(21) $y(x) = x(x+2)^2(x-1)^2$
(22) $z(x) = (x+4)^2(3x-5)^2$
(23) $a(x) = (x-1)^3(x^2+2x-5)$
(24) $b(x) = -3x(x+4)^3(x-3)$
(25) $c(x) = (-x+2)^2(3x+4)^2(x-5)$
(26) $d(x) = x^3(x+3)^2(x^2+x+2)$