CCGPS Adv Alg – 210: Multiplicity of Roots Name: ____________________________

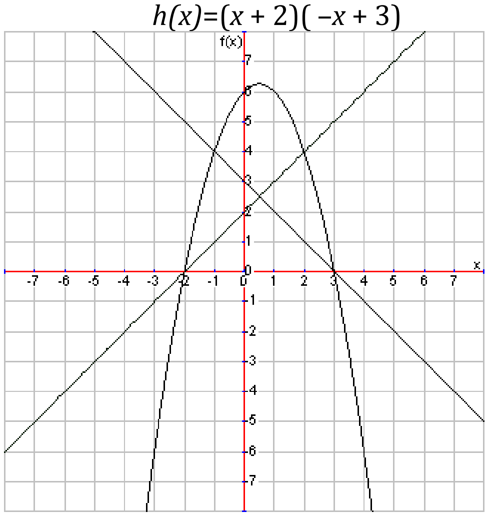
Previously, we have seen that finding the *x*-intercepts of factors of a polynomial is often easier than solving for the *x*-intercepts of the polynomial itself. For example, it is much harder to solve for the roots of $f(x) = x^4 - x^3 - 7x^2 + x + 6$ than $f(x) = (x+2)(x+1)(x-1)(x-3)$, which is the factored form of *f*. From the factored form, we can simply find the *x*-intercepts of each factor, like so:

$$\begin{array}{llll} x+2=0 & x+1=0 & x-1=0 & x-3=0 \\ x=-2 & x=-1 & x=1 & x=3 \end{array}$$

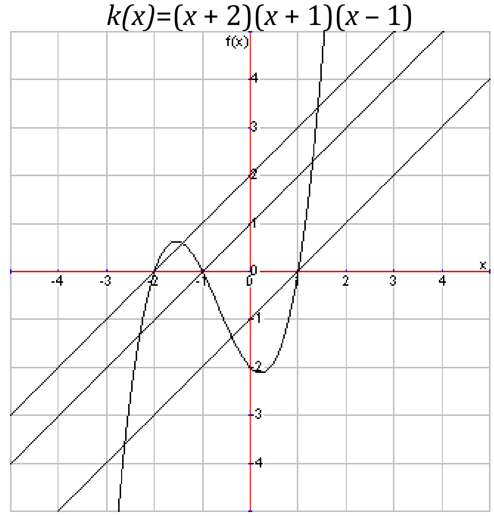
So –2, –1, 1, and 3 are the roots of *f(x)*. Graph it in your calculator if you don't believe me…

Today we will expand our understanding of the effects of factors, focusing on what happens if a factor appears multiple times. Let's compare our examples from before…

h(x)=(*x* + 2)(–*x* + 3)

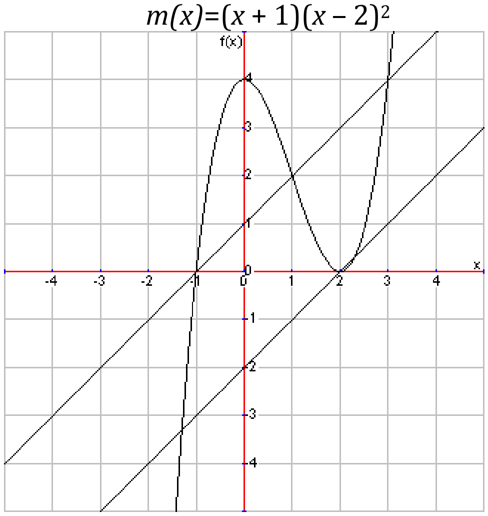


k(x)=(*x* + 2)(*x* + 1)(*x* – 1)

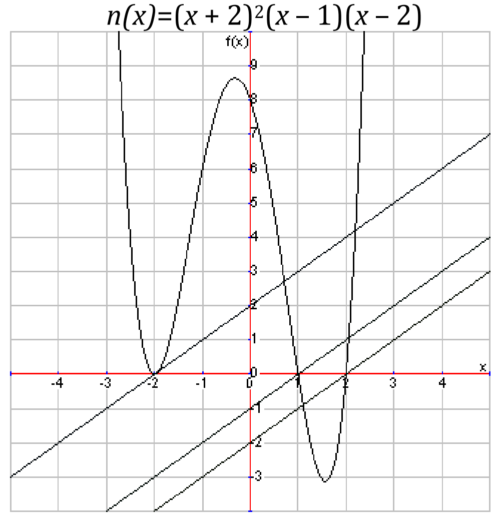


to some examples of other functions…

m(x)=(*x* + 1)(*x* – 2)²

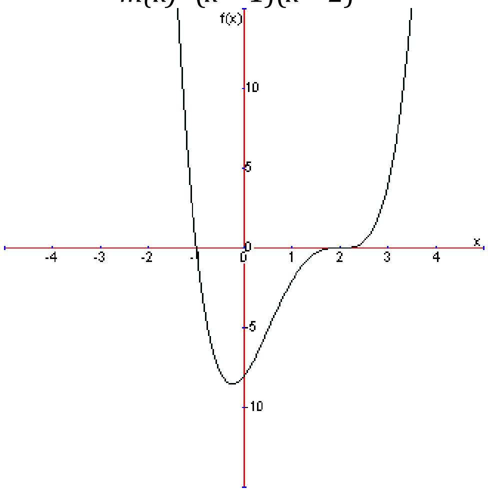


n(x)=(*x* + 2)²(*x* – 1)(*x* – 2)

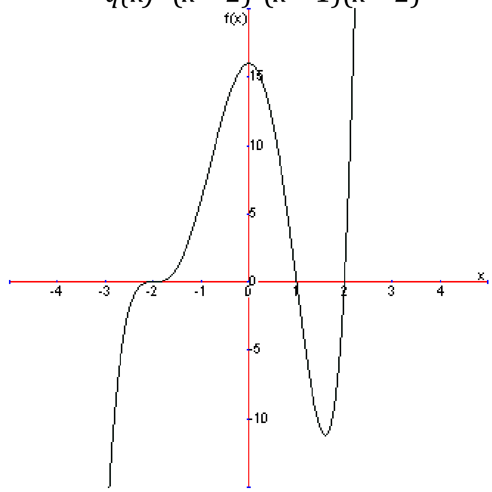


1. How does the graph behave differently at a root if the corresponding factor is squared?

$m(x)=(x+1)(x-2)^3$



$q(x)=(x+2)^3(x-1)(x-2)$



2. How does the graph behave now at a root if the corresponding factor is cubed?

3. Can you hypothesize how a graph would behave at a root if the factor was raised to the fourth power?

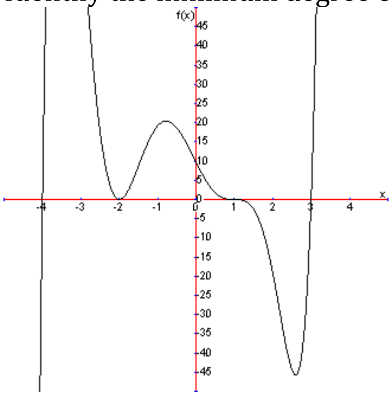
When factors are raised to a power, it is called the **multiplicity** of the root. This is because if the factor has an exponent, it is occuring in the product multiple times.

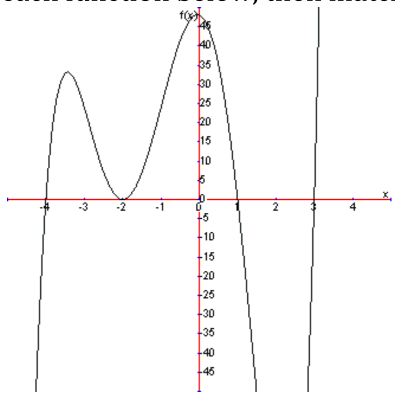
4. Summarize what you have learned about how the multiplicity of a root determines the shape of the graph at the root.

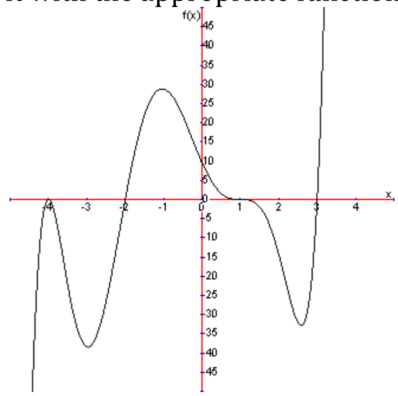
5. How can you determine the minimum degree of a polynomial based on the number of roots?

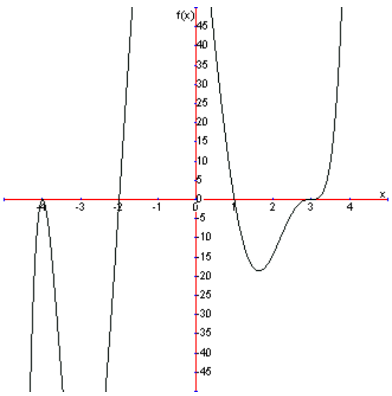
 Name: ______________________

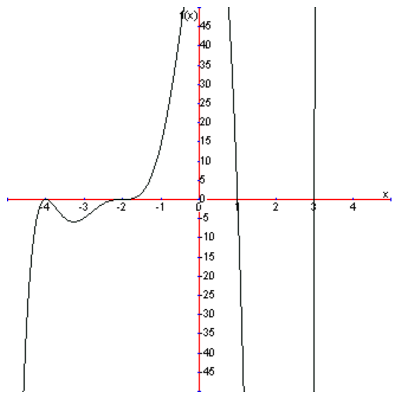
Identify the minimum degree of each function below, then match it with the appropriate function:

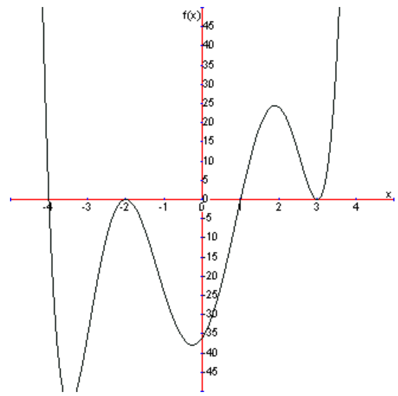


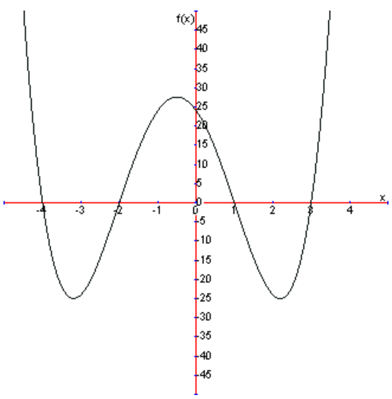


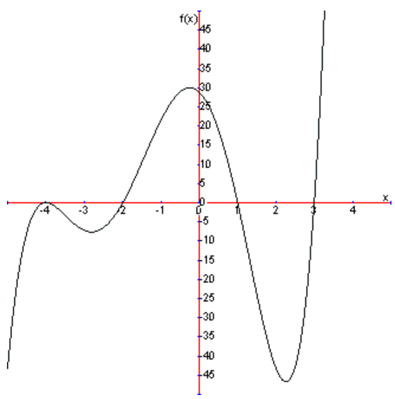


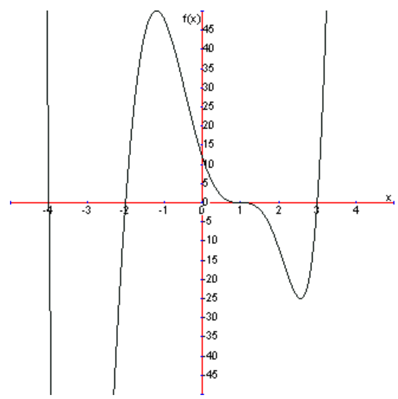


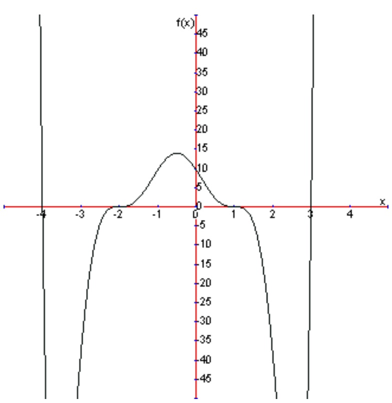












$a(x) = (x+4)(x+2)(x-1)(x-3)$

$b(x) = (x+4)(x+2)^2(x-1)(x-3)$

$c(x) = (x+4)(x+2)^2(x-1)(x-3)^2$

$d(x) = (x+4)(x+2)(x-1)^3(x-3)$

$e(x) = (x+4)(x+2)^2(x-1)^3(x-3)$

$f(x) = (x+4)^2(x+2)(x-1)(x-3)$

$g(x) = (x+4)^2(x+2)(x-1)^3(x-3)$

$h(x) = (x+4)^2(x+2)^3(x-1)(x-3)$

$j(x) = (x+4)^2(x+2)(x-1)(x-3)^3$

$k(x) = (x+4)(x+2)^3(x-1)^3(x-3)$