We are going to focus on two things today. One is a new concept and one is an old skill that needs to be brushed up. The concept is the last of our factoring techniques, and it is called "U-substitution."

Let's look at a situation where U-substitution is an effective factoring technique:

Solve for the roots of $x^4 - 7x^2 + 10 = f(x)$

Our initial reaction when see this problem should be "Ooo I really wish the function was actually $x^2 - 7x + 10 = f(x)$, because I can factor that one." Well, u-substitution is a method that essentially allows you to rewrite the function. <u>As you practice and get good at it, you won't usually need to write out all the steps, but for now I am going to so that I can explain.</u>

$x^4 - 7x^2 + 10 = 0$ (1)

$(x^2)^2 - 7x^2 + 10 = 0$ (2)

Let $u = x^2$

$u^2 - 7u + 10 = 0$ (3)

$(u-2)(u-5) = 0$ (4)

$u = 2, \quad u = 5$ (5)

$x^2 = 2, \; x^2 = 5$ (6)

$x = \pm\sqrt{2}, \; x = \pm\sqrt{5}$ (7)

So in Step (1), we need to set the function equal to zero to solve for the roots. In Step (2), the only difference is that we have rewritten x^4 as $(x^2)^2$, which as you know is a perfectly legitimate change. The only thing that prompted that change is that I recognized that I can use U-substitution to solve this problem (we'll talk about how in a second). Step (2) is basically the setup for the U-substitution that happens in Step (3). Steps (4) and (5) are just simplifying and solving. In Step (6), we essentially "undo" our previous substitution because we aren't trying to solve for *u*; we want to know *x*.

Cues that tell you to use U-substitution:

* Probably only 3 terms (can't group)
* Highest degree will be twice the middle degree. (*u* should equal the variable part of middle term)

Solve for all of the roots of the functions below. SIMPLIFY ANY RADICALS (if you don't remember how to do this, flip over to the back).

1. $x^4 - 9x^2 + 20 = f(x)$

2. $x^4 - 9x^2 + 8 = f(x)$

3. $2x^4 - 11x^2 + 5 = y$

4. $x^4 + 16x^2 - 36 = f(x)$

Complex Roots

#4 brings imaginary or complex roots into the picture. The big thing to remember is that you can always simplify $\sqrt{-1}$ to be *i*, so as long as you can simplify radicals, you should be able to handle complex roots algebraically. An important note is that complex roots don't show up on a function's graph. For example, a cubic function that has 1 real root and 2 complex roots will only cross the *x*-axis once.

Simplifying Radicals

This is as easy as being able to identify perfect square factors of a number. You want to find perfect square factors because the square root of the factor will be an integer. Remember you can apply the product property of radicals to help you simplify, like in Step (2) below.

$$\sqrt{-45} \rightarrow \sqrt{-1\cdot 9\cdot 5} \rightarrow \sqrt{-1}\cdot\sqrt{9}\cdot\sqrt{5} \rightarrow i\cdot 3\cdot\sqrt{5} \rightarrow 3i\sqrt{5}$$

Simplify the radicals below

5. $\sqrt{80}$
6. $\sqrt{-75}$
7. $\sqrt{-108}$
8. $\sqrt{1600}$

Solve for all zeros of the functions below. Remember to check for a GCF first. Any factoring method may be used. Use Quadratic Formula for quadratics that cannot be factored.

9. $3x^4 - 2x^2 + 5 = f(x)$
10. $x^3 + 9x^2 + 4x + 36 = f(x)$
11. $8x^4 - 8x^3 + 27x - 27 = f(x)$
12. $x^4 + 9x^2 + 14 = f(x)$
13. $5x^4 + 13x^2 + 6 = f(x)$
14. $3x^4 - 12x^3 + 5x^2 - 20x = f(x)$
15. $x^6 - 9x^3 + 8 = f(x)$
16. $x^3 - 6x^2 - 4x + 24 = f(x)$
17. $10x^2 + 3x - 4 = f(x)$
18. $x^8 + 5x^4 - 36 = f(x)$
19. $x^5 - 5x^3 - 24x = f(x)$
20. $x^4 - 3x^3 + 8x - 24 = f(x)$