CCGPS Adv Alg – 207: Factoring Special Cubics Name: _____

Our primary tool for factoring cubic expressions is grouping, but there are two specific forms that require another technique and, thankfully, are easy to spot. These are the sum or difference of cubes…

Sum of Perfect Cubes: $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

Difference of Perfect Cubes: $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

First thing: To be able to identify when you should use these patterns, you need to be able to recognize a perfect cube. Obviously, if the base is a variable, it is easy… something like x^3, a^3, etc. If the base is a number, you need to remember your perfect cubes. I highly recommend you memorize these values.

x	1	2	3	4	5	6	7	10
x^3	1	8	27	64	125	216	343	1000

Second thing: It should be easy to memorize the different terms; the only things that change are the signs. Notice the patterns in the signs. The first sign matches the sign of the sum or difference expression. The next sign is the opposite sign. The last sign is always positive because that term is squared.

Factor the expressions below and then we will do them as a class.

1. $x^3 + 8$
2. $x^3 - 125$
3. $64 + 27x^3$
4. $1 - x^6$

We also need to start discussing the whole point of factoring: easily solving polynomial equations. Similar to when you solved quadratics in Math 2 (because quadratics are a type of polynomial), the polynomial must equal zero before you can factor and solve. Once you factor the polynomial, you can use the same technique to solve as you do for finding the roots when you sketch graphs.

Sometimes you will end up with an expression that cannot be factored further. If it is a quadratic, you can use the quadratic formula. Otherwise, you would have to use your graphing calculator to check the rest of the roots. Since the point is to practice factoring, I'll try to make sure the practice problems are factorable.

Solve these polynomial equations or find the roots of the function. In some cases, it may help to factor out a GCF first, and you may need to use multiple techniques to factor completely.

5. $x^4 + 5x^3 - 8x - 40 = f(x)$
6. $-x^5 - 4x^4 + 27x^2 + 108x = f(x)$

7. $x^4 - 4x^3 + 8x - 32 = f(x)$

8. $343x^3 - 64 = f(x)$

9. $x^4 - x = f(x)$

10. $x^4 - x^3 - 8x + 8 = f(x)$

11. $125x^4 + 250x^3 - 8x = 16$

12. $9x^3 + 4x^2 - 5x = f(x)$

13. $x^3 - 5x^2 - 121x + 605 = f(x)$

14. $4x^4 - 12x^3 + 5x^2 = f(x)$

15. $2x^3 = 250$

16. $3x^3 + 3000 = f(x)$

Sketch any 3 of the functions above.