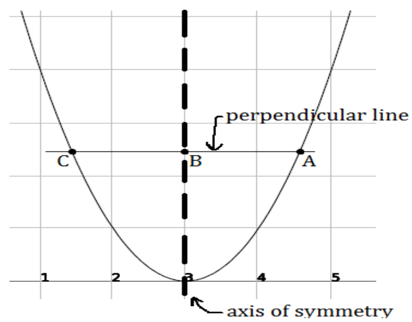
CCGPS Adv Alg – 204: Function Symmetry Name: ______________________

MM3A1c. Students will determine whether a polynomial function has symmetry and whether it is even, odd, or neither.

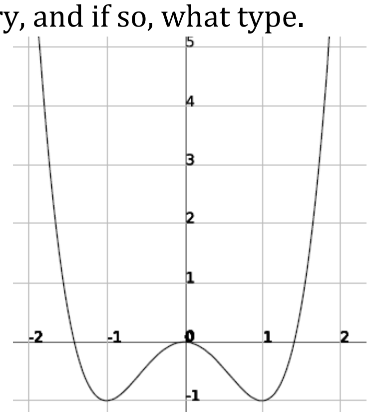
You should remember from previous studies, or from your observant nature, that symmetry is a common occurrence in our lives. Symmetry has been used throughout history. It shows up in architecture and city planning, religious and corporate symbolism, music, literature, etc. You also see symmetry in nature. Your face, hopefully, is an example. In mathematics, we focus our discussion on two common types of symmetry: _______________ and ________________ (also known as _________________).

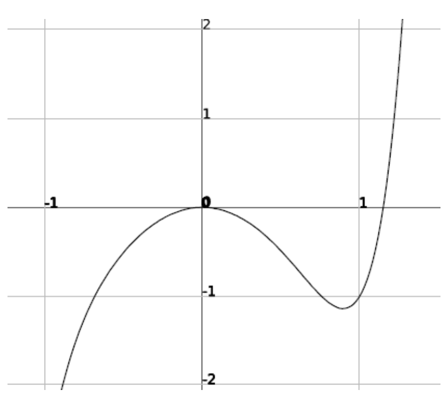
Objects with line symmetry have an axis of symmetry, like you learned to find for a parabola in Analytic Geometry. The axis of symmetry cuts the object into two reflected halves. The actual definition of line symmetry is that for every point on one side of the axis of symmetry, there is a reflected point on a line perpendicular to the axis that is the same distance from the axis of symmetry. That is easier to explain with the picture at the right. So, by our definition, if this parabola has line symmetry, then the distance between points A and B is the same as the distance between B and C. That is true no matter where the perpendicular line is. It could be at y = 2.5 like in the picture, or at y = 1, y = 5, y = anything, and the distance between A and B will be the same as the distance from B to C.

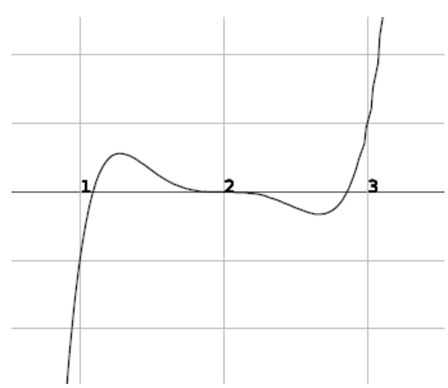


Objects with rotational symmetry look the same after an amount of rotation less than 360°. Some objects look the same many times before rotating 360°; the number of times you arrive at the same image is the *order*. For example, the Mercedes-Benz logo has rotational symmetry of order 3. The Star of David has rotational symmetry of order 6. Mathematically, there is not an easy explanation like for line symmetry, so that is basically all of the discussion we need for rotational symmetry.

Now let's take this discussion of symmetry back into the context of this unit and our graphical investigation of higher order polynomials. Determine if these functions have symmetry, and if so, what type.

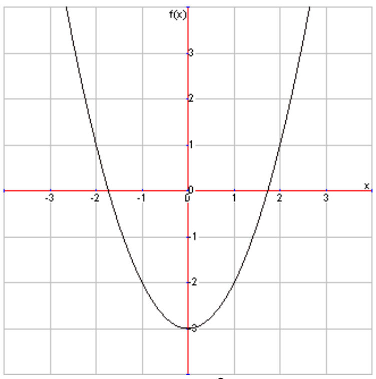




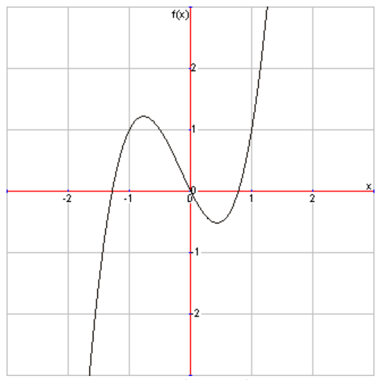


In math, we give special "recognition" to *functions that have line symmetry over the y-axis (meaning, the y-axis is the axis of symmetry)*. These are called **even functions**. Notice that it is possible for a function to have line symmetry without being an even function; that could be when the line of symmetry is somewhere other than the y-axis. Another special case that is a little more difficult to spot graphically is when *a function has rotational symmetry about the origin*. These functions are called **odd functions.** The reason I say these are more difficult is because you have to be able to correctly identify the center of rotation (like the eye of a hurricane).

Identify whether the functions below are even, odd, or neither.

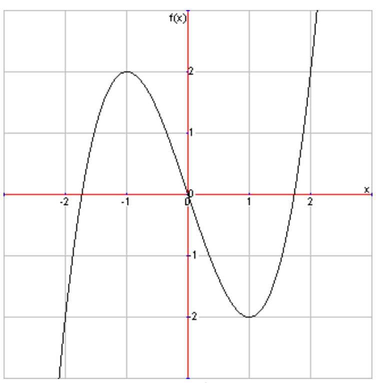


$f(x) = x^2 - 3$

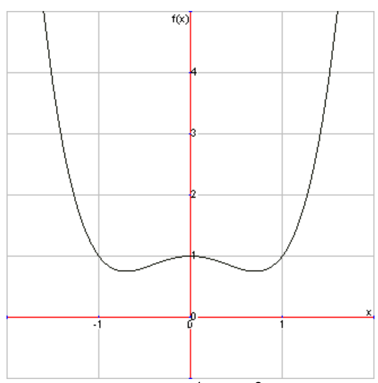


$f(x) = 2x^3 + x^2 - 2x$

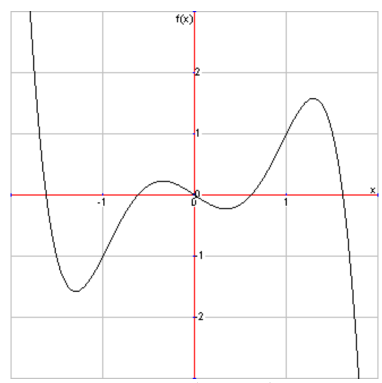
$f(x) = -x^6 - x^5 + x^4 + x^3 + x^2$



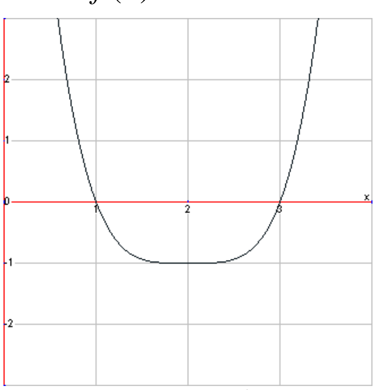
$f(x) = x^3 - 2x$



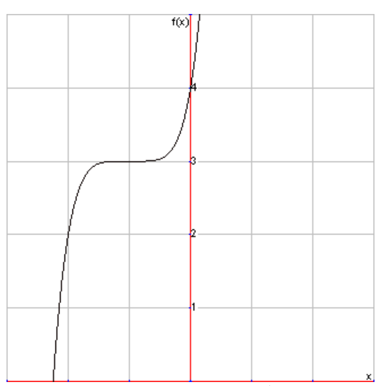
$f(x) = x^4 - x^2 + 1$



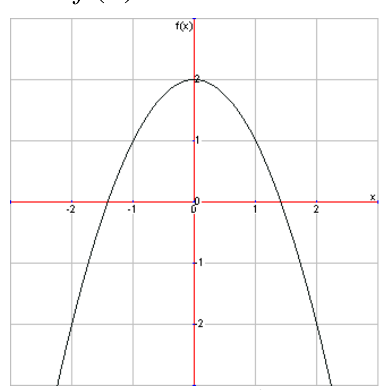
$f(x) = -x^5 + 3x^3 - x$



$f(x) = (x-2)^4 - 1$



$f(x) = (x+1)^5 + 3$



$f(x) = -x^2 + 2$

Challenge Question: Is it possible for a function to be both even and odd? **Why?**

Even Functions

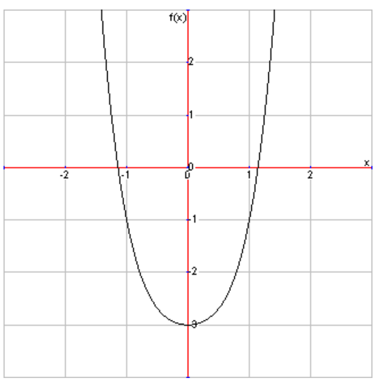
There is an algebraic way to determine whether functions are even or odd, and it involves transformations on the coordinate plane. Let's start with an easy, even function: $f(x) = x^2$. You all know that if you make a table, you can see the symmetry in the values…

x	-3	-1	0	1	3
f(x) = x^2					

but look more carefully for a moment at the pairs of points that are reflections.

1. Do you notice any patterns in the *x*- and *y*-coordinates of these points?

This leads us to the algebraic method for determining whether a function is even. Hopefully you noticed that no matter what *x* value you have in your table, –*x* has the same *f(x)* value. In words, this means that if you plug –*x* into your function for *x*, *f(x)* should remain the same if the function is even. Watch…



You can see from the graph at the left that $f(x) = x^4 + x^2 - 3$ is an even function. So let's see what happens when we substitute –*x*.

$$f(-x) = (-x)^4 + (-x)^2 - 3$$
$$f(-x) = x^4 + x^2 - 3$$
$$f(-x) = f(x)$$

This is an algebraic way to prove a function is an even function.

Odd Functions

Similarly, we can algebraically prove a function is odd. Again, let's use a basic, odd function: $f(x) = x^3$. This time when we make a table, the transformation will be a little more difficult to spot,

x	-3	-1	0	1	3
f(x) = x^3					

but look again at the pairs of reflections.

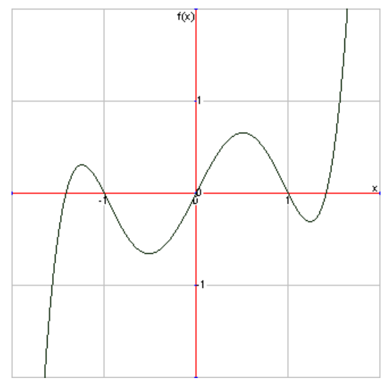
2. What is the relationship between these points this time?

Again there is a pattern, but this time hopefully you noticed that –*x* has the –*f(x)* value. Translated, if you substitute –*x* into the function for *x*, *f(x)* becomes negative. Before we do an example, let's quickly review our rules for simplifying functions so we understand what –*f(x)* means. Let's use $f(x)=x^2+x+1$.

$$f(x)=x^2+x+1$$
$$-f(x)=-(x^2+x+1)$$
$$-f(x)=-x^2-x-1$$

3. What is the difference between *f(x)* and –*f(x)*?

So, an example of an odd function is $f(x)=x^5-3x^3+2x$, and you should be able to tell from the graph at the left that it is odd.



4. We already know it is odd, so what should we get when we do the algebraic test by substituting –*x*?

5. Do the substitution and see if you get your answer to #4. Is *f(x)* an odd function?

(6 – 12) Test these functions algebraically to determine if they are even, odd, or neither.

6. $f(x)=-x^6+5x^2+3$

7. $f(x)=3x^3+2x-1$

8. $g(x)=3x^4+x^2-8$

9. $f(x)=2x^6+3x^5-4x^3$

10. $h(x)=x^5-3x^2+x-7$

11. $j(x)=3x^5-x$

12. $b(t)=2t^4-5t^2+2$

13. $c(n)=4n^7-5n^3+1$

14. $f(x)=x^{100}+x^{72}-2$

Challenge Question: Why are these functions called even and odd functions?