Our discussions the last couple days (domain & range, function composition) have really been leading up to today's topic, which is a biggie. Some functions have an

inverse function: *a function obtained by expressing the dependent variable of a function as the independent variable of another*.

Inverse functions have many uses. Mathematically, sometimes an equation is easier to solve through its inverse (exponential and logarithmic functions later this semester, for example) and you will use inverse trigonometric functions *a lot* next year. In real life, sometimes relationships are more easily considered by looking at the inverse function. An example of this could be the formula for volume of a cylinder $V = \pi r^2 h$. In the formula, volume is dependent on the radius, but maybe you would rather know what radius you would have to make a can to achieve a certain volume. In this case, you know the volume and you want to calculate the radius. That would be using the inverse of the volume formula.

Numerically

Let's look at finding inverse functions, starting with basics. Here is a function, one we used Thursday:

{(4, 2), (5, –3), (7, 1), (3, –6)}

Review the definition for inverse functions… *expressing the dependent variable of a function as the independent variable of another (function)*.

1. Identify the independent and dependent variables in our first example (which is the *x*-coordinate and which is the *y*-coordinate?).

2. The first point in our function is (4, 2). At this point, ____ is the value of the independent variable and ____ is the value of the dependent variable. In the inverse function, where the dependent variable becomes the independent variable, ____ would be the value of the independent variable and ____ would be the value of the dependent variable.

3. What is the inverse function of our example function? Is this answer a function?

4. How does the domain and range of the initial function compare to the d&r of its inverse function?

5. Find the inverse of the function {(4, 2), (5, –3), (7, 2), (3, –6)}. Is this answer a function?

Uh oh. That inverse is not a function! This leads us to VIP #1 about inverse functions: **not all functions have an inverse.** In the next section, we will find out why this is (but if you remember the difference between these two example functions that we talked about on Thursday, you already know the reason why.)

Graphically

Ok, we need an easy example to investigate inverse functions graphically. Let's use $f(x)=x^2$.

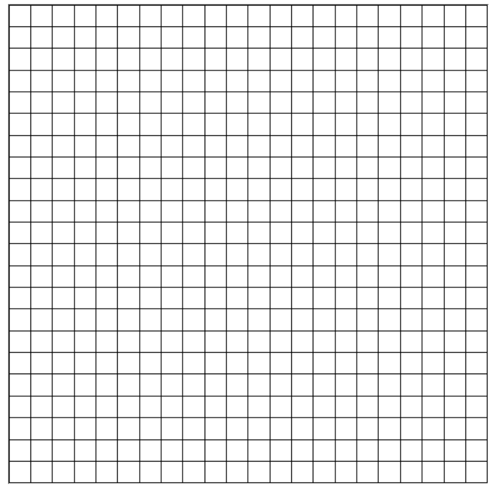
6. Fill in the tables below to get a few points and then plot the inverse function.

$f(x)$

x	y

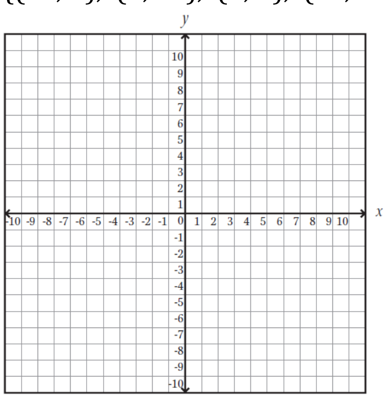
$f^{-1}(x)$

x	y

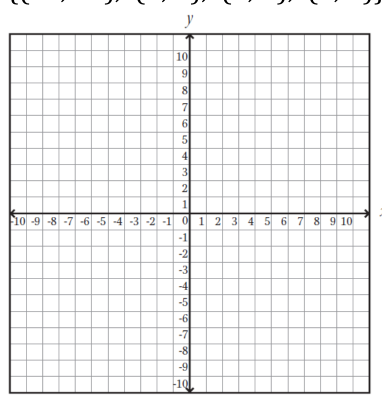


Try this with a few more function graphs and see what the outcomes look like. (Maybe graph the original function with dots and the inverse function with x's or something.)

7. $\{(-4, 2),\ (5, -3),\ (7, 3),\ (-1, -6)\}$

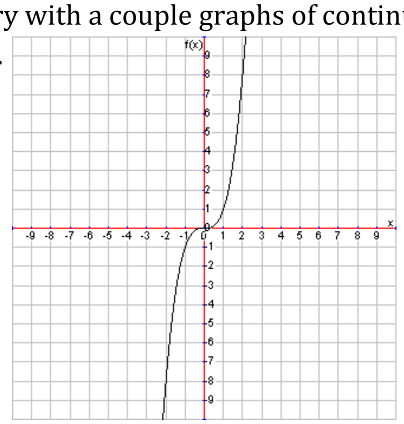


8. $\{(-1, -1),\ (0, 0),\ (2, 2),\ (3, 5)\}$

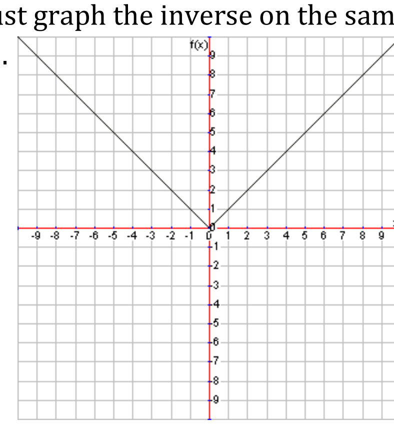


Now try with a couple graphs of continuous functions. Just graph the inverse on the same grid.

10.



11.



12. So what is the big point graphically?

Algebraically
We've talked about finding the inverse of a discrete relation/function, and how the graphs of a function and its inverse are related. But what about if we need an equation to describe the inverse function? How do we find the inverse of an equation?

Let's start with $y = 4x - 7$. Remember, the definition of the inverse function is when the dependent variable is expressed as the independent, so we want to switch the roles of our variables. So in the inverse function, the position of the *y* becomes the independent variable, and the position of the *x* becomes the dependent variable. We're going to need to rearrange the resulting equation…

	$y = 4x - 7$	Initial function
a.	$x = 4y - 7$	We are switching the roles, so typically the first step is to switch the variables to represent this switch of roles.
b.	$x + 7 = 4y$	Start solving the equation for *y*.
c.	$\frac{x+7}{4} = y$	Divide to finish solving.

Notice that we really are just solving the equation for *y*, so as long as your algebra skills (skillz?) are strong, you should be fine…

Determine the inverse of these functions.

13. $g(x) = -4x + 1$

14. $f(x) = \frac{7x + 18}{2}$

15. $f(x) = \frac{2}{3}x + 9$

16. $f(x) = \frac{3x - 2}{5}$

17. $f(x) = 3x^2 - 7$

18. $f(x) = \frac{1}{x} - 2$

19. $f(x) = -2x^3 + 1$

20. $f(x) = 3\sqrt{x} - 2$

There is one more part for discussion of inverse functions. There is a way, other than rearranging one of the equations, to check if two functions are inverse functions. We can accomplish this using function composition.

21. Confirm that $g(x) = \frac{x+3}{10}$ is the inverse function of $f(x) = 10x - 3$ by solving for $f^{-1}(x)$.

22. Determine $f(g(x))$ and $g \circ f(x)$. Observations?

23. Choose any 3 functions from questions 13 – 20 and confirm that you correctly found the inverse function using composition.