

Yesterday, we reviewed function notation. As you recall, in the function

$$f(x) = 5x + 8,$$

f represents the function name, and x is the variable representing the input of the function. $5x + 8$ tells you the operations to perform to the input in order to get the output. You have had plenty of practice evaluating functions when the input is a real number, like 2.

$$f(2) = 5 \cdot (2) + 8 = 18$$

It is also possible for the input to be an expression. In this case, you won't be able to calculate the value of the output, but just simplify down to the 'true' expression to calculate the output. For example,

$$\begin{aligned} f(x) &= 5x + 8 \\ f(2x - 3) &= 5(2x - 3) + 8 \\ &= 10x - 15 + 8 \\ &= 10x - 7 \end{aligned}$$

Practice working with the functions below, simplifying fully and evaluating where possible.

1. $a(x) = 4x - 3$

$$a(3) =$$

2. $f(x) = 2x + 15$

$$f(x + 4) =$$

3. $g(x) = -3x + 1$

$$g(-2x + 9) =$$

4. $b(x) = x^2 + 2x + 3$

$$b(4) =$$

5. $t(x) = x^2 - 3x + 2$

$$t(2x + 1) =$$

6. $f(x) = x^3 - 1$

$$f(x - 4) =$$

Ok, here comes the jump into today's topic. We've discussed how to work with functions when the input is a real number or an expression. How about when the input is another function?

7. $f(x) = 10x - 7$, $g(x) = 2x + 1$

$$f(g(x)) =$$

The process of function composition is very similar to substituting an expression into the function.

$f(g(x))$ can also be written $(f \circ g)(x)$ and is read 'f of g of x' or 'f composed with g.' The only big, helpful hint for function composition is to start from the inside and work your way out. For example,

Ex: If $f(x) = 10x - 7$, $g(x) = 2x + 1$, find $g(f(2))$.

To find g of f of 2, start inside out. $f(2) = 10(2) - 7 = 13$. Substituting this into our original statement means we now want to know $g(13)$, which equals $2(13) + 1 = 27$.

8. If $f(x) = 2x + 7$ and $g(x) = 3x - 2$, find $f(g(6))$.
9. If $f(x) = 2x + 7$ and $g(x) = 3x - 2$, find $(g \circ f)(6)$.
10. If $f(x) = -5x + 2$ and $g(x) = \frac{1}{2}x + 4$, find $(f \circ g)(12)$.
11. If $g(x) = -3x^2 + 6$ and $h(x) = 9x + 3$, find $g(h(\frac{1}{3}))$.
12. If $f(x) = 4x - 7$ and $g(x) = |2x - 9|$, find $(f \circ g)(-5)$.
13. If $f(x) = 4x - 7$ and $g(x) = |2x - 9|$, find $(g \circ f)(-5)$.
14. If $f(x) = 3|x - 4| + 6$ and $h(x) = -x^3$, find $f(h(4))$.
15. If $f(x) = 3|x - 4| + 6$ and $h(x) = -x^3$, find $h(f(4))$.
16. If $g(x) = -2x^2 - 5x$ and $h(x) = 3x + 2$, find $g(h(x))$.
17. If $f(x) = x^2$ and $g(x) = -12x + 7$, find the domain and range of $f(x)$, $g(x)$, and $f(g(x))$.
18. Is function composition commutative? [Review your answers to some questions above...]